STOCK WARS: INVENTORY COMPETITION IN A TWO-ECHELON SUPPLY CHAIN WITH MULTIPLE RETAILERS

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This paper studies the competitive and cooperative selection of inventory policies in a two-echelon supply chain with one supplier and N retailers. Stochastic demand is monitored continuously. Retailers incur inventory holding and backorder penalty costs. The supplier incurs holding costs for its inventory and backorder penalty costs for backorders at the retailers. The latter cost reflects the supplier's desire to maintain adequate availability of its product to consumers. Previous research finds the supply chain cost minimizing reorder point policies, the cooperative solution. The competitive solution is a Nash equilibrium, a set of reorder points such that no firm can deviate from the equilibrium and lower its cost. It is shown that Nash equilibria exist and a method is presented to find all of them. In some settings the cooperative solution is a Nash equilibrium; competition does not necessarily lead to supply chain inefficiency. In other settings, competition leads to costs that are substantially higher than optimal. Usually (but not always), the competitive supply chain carries too little inventory. Three cooperation strategies are discussed: change incentives, change equilibrium, and change control. A set of contracts is provided that changes the firms' incentives so that the optimal policy is a Nash equilibrium. An equilibrium change can improve performance but does not guarantee optimal performance. To change control, the firms let the supplier choose all reorder points, a key component in vendor managed inventory. That change leads to optimal supply chain performance.

1. INTRODUCTION

This paper studies competitive behavior in a supply chain with one supplier and N retailers facing stochastic demand. The supply chain's objective is to minimize inventory holding plus backorder penalty costs. Retailers wish to minimize their own holding and backorder penalty costs. The supplier wants to minimize its holding cost plus its backorder penalty cost charged at the retailer level. This latter cost reflects the supplier's desire to maintain adequate availability of its product to consumers. Given these preferences, will the players choose policies that achieve the supply chain's objective?

There are several explanations for why choices might deviate from the system optimal solution. Players may not be able to evaluate optimal policies. They may lack the necessary information to find optimal policies. (See Anand and Mendelson 1997 for a model in which that is the case.) Alternatively, players may *knowingly* deviate from the optimal policy because they can privately benefit from a deviation. This paper explores that explanation; each player possesses all of the information needed to evaluate optimal policies (in the class of policies considered), so every player could choose the optimal policy but may not do so for personal gain.

Because cooperation requires effort (e.g., management time, accounting systems to track incentives and compliance, etc.), it is useful to know the expected returns from cooperation before committing to cooperative activities. An understanding of supply chain inventory competition is needed to evaluate those potential returns. Furthermore, by understanding how competition leads to operating inefficiency, it is possible to recommend productive cooperative strategies. For instance, if it is discovered that with competition a firm chooses too little inventory, then cooperation should focus on strategies to encourage that firm to carry more inventory, and not the reverse.

This paper studies the supply chain inventory (SCI) game. In the SCI game, the firms manage inventory with reorder point policies. (The policies are described in detail in §2.) Competition leads to a pure strategy Nash equilibrium in reorder points, which is a set of reorder points such that no player can lower its cost by deviating from the equilibrium, assuming the other players play their equilibrium strategies; there are no profitable unilateral deviations. It is shown that the SCI game belongs to a special class of games, called *supermodular games.* As a result, there exists at least one pure strategy Nash equilibrium. Further, in the SCI game, the firm's strategies (reorder points) are strategic substitutes: As the supplier raises its reorder point, the retailers will not increase (and often decrease) their reorder points. (See Bulow et al. 1985, for a discussion of strategic substitutes and complements.) Finally, the special structure of supermodular games facilitates the search for the set of Nash equilibria.

Axsäter (1993) evaluates and finds reorder point policies in the SCI game that minimize total supply chain costs, i.e., the optimal policies. In a numerical study, this paper shows that the optimal reorder points are sometimes a Nash equilibrium. In those settings, firms can choose the optimal reorder points and be confident that the other players will do so too. Hence, competition does not necessarily lead

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to supply chain inefficiency. In other settings, the optimal policies are not a Nash equilibrium, but there is only a modest percentage cost increase between the Nash equilibrium cost and the optimal cost, i.e., a small competition penalty. This tends to occur when the supplier and the retailers incur backorder costs at approximately the same rate. However, when either the supplier or the retailers incur backorder costs at a significantly higher rate than the other players, the Nash equilibria often lead to substantial supply chain inefficiency, e.g., competition penalties up to 1400%. It is concluded that the competition penalty is context specific or, put simply, the value of cooperation is sometimes small and sometimes enormous.

Interestingly, total supply chain inventory tends to be lower in a Nash equilibrium than in the optimal solution. In those cases, cooperating to choose the optimal policies would mean that the firms would *increase* supply chain inventory. With competition, supply chain inventory often acts like a public good; everybody wants more of it, but nobody wants to pay for it. However, there are cases in which competition leads to more inventory than the supply chain needs (up to 12 times more in the sample considered). Those cases tend to occur when the supplier cares little about customer service, so the supplier carries little inventory. As a result, the poor retailers must cope with very long lead times; they may carry more inventory than the entire system does with the system optimal solution.

Three cooperation strategies are discussed. The first strategy is to change the players' incentives so that the optimal policies are a Nash equilibrium. This is done with a contract that creates two transfer payments: a payment from the retailers to the supplier based on the retailers' backorders; and a payment from the supplier to the retailers for the supplier's backorders, i.e., for late deliveries. The second strategy is the simplest. If there are multiple Nash equilibria, then the firms should make sure they choose the lowest cost equilibrium. This will not guarantee optimal performance, but it requires a minimal change to current operating procedure. The third strategy transfers to the supplier the responsibility for choosing policies at all locations in the supply chain. This is a major component of the vendor managed inventory (VMI) programs that many companies have recently established. It is found that VMI coordinates the supply chain (optimal policies are chosen) as long as the firms are willing to use fixed transfer payments so that they can share the gains from VMI.

The next section details the model studied. The subsequent section reviews the related literature. Section 4 presents the analysis of the SCI game, and §5 discusses cooperation strategies. Section 6 details the numerical study. The final section concludes.

2. THE SUPPLY CHAIN INVENTORY GAME

A single supplier sells a single product to N independent identical retailers. (The assumption of identical retailers can be relaxed in some cases, as discussed in §4.5.) Because the retailers are identical, an "r" subscript identifies a variable or parameter that applies to a single retailer. The supplier is the retailers' only source of inventory. The supplier operates the central warehouse for this supply chain, so the subscript "w" identifies the supplier. (This also provides some consistency with the notation in Axsäter 1993.)

Each retailer experiences independent Poisson demand with mean λ_r. The supplier's source has infinite capacity. The supplier's replenishments always arrive L_w time units after they are ordered. Once inventory is shipped to a retailer, it is received L_r time units later. Each firm continuously monitors its inventory position, which equals on-hand inventory plus on-order inventory minus backorders. If inventory is available, the supplier ships a retailer's order immediately, i.e., retailer orders are filled with a first-come-first-serve basis. The supplier's backorders are unshipped retailer orders. All demands are backordered, hence there are no lost sales.

Retailers implement (R_r, Q_r) ordering policies, i.e., they order Q_r units whenever their inventory position equals R_r. Call Q_r units a subbatch. Because retailer orders always equal an integer multiple of Q_r units, all supplier variables are measured in subbatches. To replenish its inventory, the supplier implements an (R_w, Q_w) ordering policy, i.e., when its inventory position equals R_w, it orders Q_w subbatches from its source. Call Q_w subbatches a batch. A batch contains $Q_w Q_r$ units.

The supplier incurs holding costs for units in its possession at rate h_w per unit, and the retailers incur holding costs per unit in their possession at rate h_r. Holding costs are not incurred for units on route to the supplier or to the retailer because the players cannot influence those costs. Each retailer incurs backorder penalty costs per backordered unit at rate $\beta_r \geqslant 0$. The supplier may also care about the availability of its product to consumers. To model this preference, the supplier incurs a cost $\beta_w > 0$ per unit backordered at a retailer per unit time. Hence, at each retailer total backorder penalty costs accumulate at rate $\beta = \beta_w + \beta_r$. Note that the supplier does not incur a penalty cost for backordered retailer orders (although that cost could easily be included into the model). However, a late retailer shipment might cause retail backorders, which do incur costs for the supplier, so supplier backorders are indirectly costly to the supplier.

Let σ_r be the set of reorder points available to a retailer, i.e., σ_r is the set of strategies a retailer can choose in this game; σ_w is analogously defined. Assume $\sigma_r \in [-Q_r, \widehat{R}]$ and $\sigma_w \in [-Q_w, \widehat{R}]$, where $\widehat{R}$ is a very large constant. The lower bound on the player's strategies is not restrictive, because no firm will ever wish to choose a lower reorder point: Decreasing the reorder point below the lower bound increases backorders but does not reduce inventory. The upper bound on the strategy space is for analytical convenience, and it too imposes no restrictions on the game: Once a reorder point is very large, backorders are essentially zero, so an increase in the reorder point only increases inventory.

In this game the firms simultaneously choose their reorder points, and then they implement the chosen policies over an infinite horizon. All costs and parameters are common knowledge to the firms. Each firm attempts to minimize its own costs, knowing that the other firms do the same.

See Appendix A for a summary of the major notation.

3. LITERATURE REVIEW

There is a substantial literature that studies reorder point policies in the SCI game. With one-for-one ordering ($Q_w = 1$ and $Q_r = 1$), Sherbrooke (1968) and Graves (1985) provide approximate evaluation of policies. With batch ordering ($Q_w > 1$ or $Q_r > 1$), approximate evaluations include Deuermeyer and Schwarz (1981), Moinzadeh and Lee (1986), Lee and Moinzadeh (1987a, b), and Svoronos and Zipkin (1988). Axsäter (1990) provides exact evaluation for one-for-one ordering, and Axsäter (1993) extends this method to provide exact evaluation for batch ordering with identical retailers. This research implements Axsäter's method, so policies are evaluated exactly and optimal reorder point policies are found. None of the above papers considers inventory competition.

Andersson et al. (1996), Chen (1999), Lee and Wang (1999), and Porteus (2000) provide incentive schemes to decentralize decision making in multiechelon supply chains with stochastic consumer demand and multiple periods, i.e., the supply chain optimal policy is a Nash equilibrium with each of their incentive schemes. They do not investigate competitive behavior before they impose their accounting schemes. In the SCI game with one-for-one ordering, Axsäter (2001) provides an accounting scheme that guarantees each player a cost that is no greater than an initial value, even if the other players choose different policies. While this scheme protects each player's cost, it does not guarantee that the players will choose the optimal policies, nor does it predict what will be the initial cost values. Instead of modifying costs, others have studied coordination of supply chain activities by imposing constraints on each member (see Muckstadt and Thomas 1980, and Hausman and Erkip 1994).

Chen et al. (2001) and Bernstein and Federgruen (1999) study inventory coordination in a two-echelon supply chain with multiple retailers and deterministic demand. Because they assume deterministic demand, it is difficult to make a meaningful comparison between their results and this work.

Several papers study inventory competition among one or more retailers or agents but not in a multiechelon setting: See Li (1992), Lippman and McCardle (1997), and Mahajan and van Ryzin (1998). In the SCI game, the retailers do not compete for customers. Competition occurs between the supplier and each retailer.

This research is similar in spirit to Cachon and Zipkin (1999). They study a two-echelon supply chain with one supplier and one retailer in which the supplier cares about consumer backorders. (See also Caldentey and Wein 1999 for a model that uses that preference structure.) They find a unique Nash equilibrium in inventory policies and demonstrate that the Nash equilibrium is (essentially) never the optimal solution. Hence, in their setting, competition in inventory policies decreases supply chain efficiency. However, a numerical study demonstrates that the competition penalty is context specific. When the supplier and retailer incur about the same backorder costs, the competition penalty is small; but when either the supplier or the retailer incur backorder costs, which greatly exceed the other player's cost, the competition penalty can be enormous.

The theory of supermodular games is used to study the SCI game. For a sample of work on supermodular games, see Topkis (1979, 1998), Vives (1990), and Milgrom and Roberts (1990). This class of games has been applied in a wide range of economic settings. In the operations literature, Lippman and McCardle (1997) apply the theory to study inventory competition between two retailers, and Cachon and Lariviere (1999) study capacity allocation among multiple retailers.

There has been some recent work on VMI systems. See Cachon and Fisher (1997) and Clark and Hammond (1997) for some empirical work. In a theoretical model, Aviv and Federgruen (1998) found that VMI is substantially more beneficial to a supply chain than merely sharing information. In a serial two-echelon supply chain with a single demand period, Narayanan and Raman (1997) show that VMI can improve performance but does not guarantee optimal performance. Fry et al. (1999) and Cheung and Lee (1998) study how VMI enables the implementation of a better inventory management policy. In this paper the same type of policy is implemented (reorder point policies) whether VMI is adopted or not.

There are other papers that investigate inventory and incentive issues in supply chains, but in significantly different settings. See Anupindi and Bassok (1998), Cachon (1998), Lariviere (1998), and Tsay et al. (1998) for summaries of the literature.

4. INVENTORY COMPETITION

This section begins with the supplier's and the retailers' cost functions. Section 4.2 demonstrates that this game is supermodular, §4.3 outlines a procedure for finding Nash equilibria, §4.4 compares the Nash equilibria to the optimal solution, and §4.5 discusses a nonidentical retailer game.

4.1. Cost Functions

It is well known that when firms implement reorder point policies, a retailer's average cost depends only on its own reorder point and R_w, i.e., a retailer's decision has no impact on the other retailers (holding R_w constant). Further, the supplier's inventory depends only on R_w. Hence, when retailers are identical there is little value to track the complete set of retailer reorder points. Therefore, for notational convenience, it is assumed that the retailers choose the same reorder point, R_r. All of the subsequent results are easily extended to allow the retailers to choose different reorder points, as is discussed in §4.5. (For a given R_w, there may exist multiple reorder points that minimize a retailer's cost. In that situation it is possible that the retailers choose different reorder points, but their costs would be identical. Accounting for that technical detail adds little value.)

When $Q_w = 1$ and $Q_r = 1$, let $c(R_w, R_r)$, $c_w(R_w, R_r)$, and $c_r(R_w, R_r)$ be expected costs per unit time for the supply chain, the supplier and a retailer respectively. Because supply chain costs are the sum of the firms' costs,

$$c(R_w, R_r) = Nc_r(R_w, R_r) + c_w(R_w, R_r).$$

Axsäter (1990) exactly evaluates $c(R_w, R_r)$. See Appendix B for the details on how to use Axsäter's results to evaluate $c(R_w, R_r)$, $c_r(R_w, R_r)$, and $c_w(R_w, R_r)$.

Some additional definitions are needed before evaluating costs with batch ordering ($Q_r > 1$ or $Q_w > 1$). Pick a customer demand that causes a retailer to order (i.e., that demand lowers the retailer's inventory position from $R_r + 1$ to R_r). and also causes the supplier to order (i.e., the retailer's order lowers the supplier's inventory position from $R_w + 1$ to R_r). Call that demand the *trigger demand*, call that retailer the *trigger retailer*, and call that retailer's order the *trigger order*. Number all subsequent demands relative to the trigger demand, i.e., demand i is the ith demand following the trigger demand. Retailer orders following the trigger order are also numbered, i.e., order j is the jth retailer order following the trigger order. Define $p_{i,j}$ as the probability demand i triggers order j, i.e., the probability that when demand i occurs at some retailer, this retailer submits order j. Define the random variable P_j to be the demand that triggers order j,

$$\Pr(P_j \leqslant i) = \sum_{k=0}^{i} p_{k,j}.$$

From Axsäter (1993), $I_j^l \leqslant P_j \leqslant I_j^u$, where

$$I_j^l = \begin{cases} j & j < N \\ N - 1 + (j - N + 1)Q_r & \text{otherwise} \end{cases}, \tag{1}$$

and $I_j^u = (N-1)(Q_r - 1) + jQ_r$. Note that P_{j+1} stochastically dominates P_j, i.e., for all i, $\Pr(P_{j+1} \leqslant i) \leqslant \Pr(P_j \leqslant i)$. Furthermore, $I_j^l < I_{j+1}^l$ and $I_j^u - I_j^l \leqslant I_{j+1}^u - I_{j+1}^l$: The earliest demand that could trigger order j cannot trigger order $j+1$, and the set of demands that can trigger order j is no smaller than the set of demands that can trigger order $j+1$.

Define $q_{m,j}$ as the probability m demands occur at the trigger retailer over the interval of time $(t_0, t_j]$, where t_0 is when the trigger order occurs and t_j is the time order j occurs. For all $m > jQ_r$, $q_{m,j} = 0$ (if demands after time t_0 only occurred at the trigger retailer, then demand jQ_r would trigger order j).

Let $C(R_w, R_r)$ be the supply chain's expected costs per unit time for any $Q_r \geqslant 1$ and $Q_w \geqslant 1$. From Axsäter (1993),

$$C(R_w, R_r) = \left(\frac{1}{Q_w Q_r}\right)\Bigg[\sum_{j=\max\{I, -R_w - Q_w\}}^{-R_w - 1} \sum_{k=R_r+1}^{R_r+Q_r} \cdot \sum_{m=0}^{jQ_r} q_{m,j} c(-1, k-m-1) + \sum_{j=\max\{0, R_w+1\}}^{R_w+Q_w} \sum_{k=R_r+1}^{R_r+Q_r} \sum_{i=I_j^l}^{I_j^u} p_{i,j} c(i-1, k-1)\Bigg]. \tag{2}$$

The above reveals that costs with batch ordering are evaluated as a convex combination of costs with one-for-one ordering. Axsäter (1993) evaluates the $p_{i,j}$ and $q_{m,j}$ probabilities, but those details are not needed for this analysis. Let $C_w(R_w, R_r)$ and $C_r(R_w, R_r)$ be the supplier's and a retailer's expected costs per unit time with any batch size. To evaluate those costs, use Equation (2) but substitute the appropriate one-for-one ordering cost function ($c_w(\cdot, \cdot)$ or $c_r(\cdot, \cdot)$).

4.2. Competitive Analysis

Milgrom and Roberts (1990) outline several conditions that are required for the SCI game to be supermodular. One condition is that each player's strategies must be ordered. Let $\sigma \geqslant \sigma'$ denote that strategy σ is higher in the order than strategy σ'. The following ordering convention is adopted for the SCI game: If $R_w \geq R'_w$, then $R_w \geqslant R'_w$, and if $R_r \leq R'_r$, then $R_r \geqslant R'_r$. Hence, the natural ordering is adopted for the supplier (a higher supplier strategy corresponds to a larger R_w), whereas the opposite ordering is adopted for the retailers (a higher retailer strategy corresponds to a smaller R_r). This ordering is cumbersome but necessary, because the SCI game is not supermodular when the natural ordering is applied to both players. Lippman and McCardle (1997) and Milgrom and Roberts (1990) study other games in which similar orderings are implemented. To help avoid confusion, the terms "higher" and "lower" refer to the ordering of the strategies, and the terms "larger" and "smaller" refer to the natural ordering.

Another condition for supermodularity is that σ_w and σ_r are complete lattices, which is easily confirmed. (Details are provided in the proof of Theorem 3.) The critical condition, with respect to the SCI game, is that the players' cost functions exhibit *decreasing differences*: Given the two lattices σ_w and σ_r, a function $f\colon \sigma_w \times \sigma_r \to \Re$ has decreasing differences in its two arguments x and y if for all $x \geqslant x'$ and $y \geqslant y'$,

$$f(x, y) - f(x', y) \leqslant f(x, y') - f(x', y').$$

(It is typically stated that the players' payoff functions must have increasing differences in their arguments. In the SCI game, the player's payoff functions are the negative of their cost functions. Decreasing differences in the cost functions imply increasing differences in those payoff functions.) Given the ordering over σ_w and σ_r, the cost functions have decreasing differences if for all reorder points,

$$C_w(R_w+1, R_r) - C_w(R_w, R_r) \leq C_w(R_w+1, R_r+1) - C_w(R_w, R_r+1), \quad (3)$$

and

$$C_r(R_w+1, R_r) - C_r(R_w+1, R_r+1) \leq C_r(R_w, R_r) - C_r(R_w, R_r+1). \quad (4)$$

Assuming the above hold, if the supplier chooses a larger reorder point (a higher strategy), the retailers will tend to choose a lower reorder point (a higher strategy, too). This is a key characteristic of supermodular games: As player i chooses a higher strategy, player j will tend to choose a higher strategy as well.

It remains to confirm Equations (3) and (4). With one-for-one ordering, decreasing differences in the firms' cost functions follows quickly from a result in Axsäter (1990). All proofs are in Appendix C.

LEMMA 1. *$c_r(R_w, R_r)$ and $c_w(R_w, R_r)$ have decreasing differences in R_w and R_r.*

Suppose $f(x, y)$ and $g(x, y)$ have decreasing differences in x. It is easy to show that their convex combination, $\rho f(x, y) + (1-\rho) g(x, y)$, has decreasing difference in x, too, assuming the probability weight ρ is independent of x and y. (See Topkis 1998, Lemma 2.6.1.) With batch ordering, Equation (2) indicates that a firm's cost is the convex combination of costs under one-for-one ordering. However, the probability weights, $p_{i,j}$ and $q_{m,j}$ are not independent of the reorder points, R_w and R_r. Therefore, when the firms implement batch ordering, demonstrating that the firms' cost functions have decreasing differences in their arguments requires taking advantage of the special structure of the $p_{i,j}$ and $q_{m,j}$ probabilities.

LEMMA 2. *$C_w(R_w, R_r)$ and $C_r(R_w, R_r)$ have decreasing differences in R_w and R_r.*

There are some remaining requirements to establish that the SCI game is supermodular (e.g., payoff continuity conditions). Details are provided in the proof of the following theorem. Because the SCI game is supermodular, it immediately follows that a pure strategy Nash equilibrium exists.

THEOREM 1. *The SCI game is supermodular and there exists a pure strategy Nash equilibrium, (R_w^*, R_r^*).*

4.3. Finding Equilibria

Finding equilibria in a supermodular game is straightforward, but additional definitions are needed. Define x_{-i} as a vector of strategies by all players but player i. A strategy x_i for player i is *strongly dominated* by another strategy x_i' for player i, if for all x_{-i} player i's cost when it plays x_i is greater than its cost when it plays x_i'. A strategy is *serially undominated* if it survives the iterative elimination of strongly dominated strategies. (More formally, define $U(\sigma)$ as the set of undominated strategies given the strategy profile σ, which is a set of feasible strategies. Define $\sigma^0 = \{\sigma_w, \sigma_r\}$ and for all $\tau \geq 1$, $\sigma^\tau = U(\sigma^{\tau-1})$. A strategy x_i is serially undominated in $x_i \in \sigma_i^\tau$ for all $\tau \geqslant 1$.) For the supplier, let $\overline{R}_w$ and $\underline{R}_w$ be the largest and smallest serially undominated strategy. For the retailers, $\overline{R}_r$ and $\underline{R}_r$ are analogously defined, where $\overline{R}_r \geq \underline{R}_r$; this relationship is with respect to the natural ordering.

THEOREM 2. *The strategy pairs $(\overline{R}_w, \underline{R}_r)$ and $(\underline{R}_w, \overline{R}_r)$ are Nash equilibria in the SCI game.*

According to the above theorem, the search for Nash equilibria can begin by eliminating dominated strategies. To see how this is done, define a player's reaction correspondence, $R_w(R_r)$ and $R_r(R_w)$ as the set of reorder points that minimize the player's costs, given the reorder point chosen by the other players:

$$R_w(R_r) = \arg\min_x C_w(x, R_r) \quad \text{and}$$

$$R_r(R_w) = \arg\min_x C_r(R_w, x).$$

Define $\underline{R}_w(R_r)$ and $\overline{R}_w(R_r)$ as the supplier's smallest and largest strategy in the set of cost minimizers,

$$\underline{R}_w(R_r) = \min(R_w(R_r)), \qquad \overline{R}_w(R_r) = \max(R_w(R_r)).$$

The corresponding functions for the retailers are analogous, and note that the natural ordering is again applied,

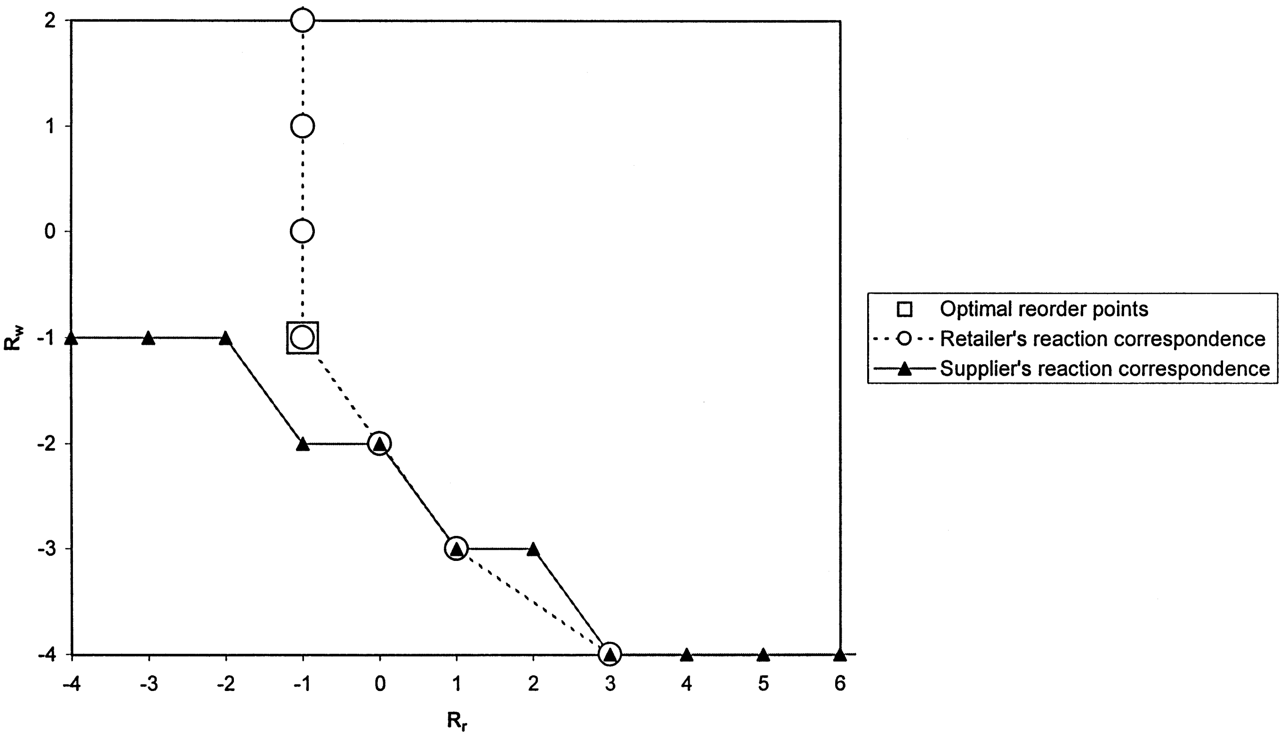
$$\underline{R}_r(R_w) = \min(R_r(R_w)), \qquad \overline{R}_r(R_w) = \max(R_r(R_w)).$$

Figure 1 displays the reaction correspondences for one of the test problems discussed in §6. In this problem the reaction correspondences have one element, so the minimum and maximum reaction functions are identical.

To find $(\overline{R}_w, \underline{R}_r)$, begin with the smallest retailer strategy, $R_r = -Q_r$. Given that the retailers will not choose $R_r < -Q_r$, the supplier can eliminate $(\overline{R}_w(-Q_r), \widehat{R}]$ from its strategy space. If the supplier won't choose $R_w > \overline{R}_w(-Q_r)$, the retailers can eliminate $[-Q_r, \underline{R}_r(\overline{R}_w(-Q_r)))$ from their strategy space. At the convergence of this process, the remaining strategy spaces will be $[-Q_w, \overline{R}_w]$ and $[\underline{R}_r, \widehat{R}]$. To illustrate with Figure 1, start with $R_r = -Q_r = -4$, which reduces the supplier's strategy space to $[-4, -1]$. Given that $R_w \leq -1$, the retailer's strategy space is reduced to $[-1, \widehat{R}]$. Given $R_r \geq -1$, the supplier will only choose $R_w \leq -2$, which in turn means the retailer will only choose $R_r \geq 0$. No further strategies can be eliminated, so $(\overline{R}_w, \underline{R}_r) = (-2, 0)$, and this is a Nash equilibrium. The process to find $(\underline{R}_w, \overline{R}_r)$ is analogous.

When $(\overline{R}_w, \underline{R}_r) = (\underline{R}_w, \overline{R}_r)$, there is a unique Nash equilibrium. When these pairs are different, there are at least two equilibria but there may be more. These additional equilibria can be found through an exhaustive search of the remaining undominated strategies. In Figure 1, $(-3, 1)$ is a third Nash equilibrium.

Figure 1. Reaction correspondences (Problem 4: $\lambda = 0.1$, $N = 4$, $\beta_r = 15$, $\beta_r = 5$, $Q_r = 4$, $Q_w = 4$).



The process to find Nash equilibria is about as computationally intensive as the process to find optimal reorder points. The search for equilibria requires evaluating both $C_r(R_w, R_r)$ and $C_w(R_w, R_r)$, rather than just $C(R_w, R_r)$, but these evaluations are not too burdensome. Evaluating the $p_{i,j}$ probabilities is computationally intensive, especially for large N and Q_r, but this evaluation is the same whether searching for Nash equilibria or the optimal solution. To find the optimal solution requires an exhaustive search over the supplier's feasible reorder points (because $C(R_w, R_r)$ is not necessarily jointly convex in R_w and R_r), but the elimination of dominated strategies may allow for a smaller search. (Interestingly, $-C(R_w, R_r)$ is not supermodular, so the theory of supermodular functions does not help to find the centralized solution.)

4.4. Nash Equilibria and the Optimal Solution

Given that Nash equilibria exist, how do choices in these equilibria differ from optimal reorder points (R_w^o, R_r^o)? To make this comparison, define $R_r^o(R_w)$ as the set of retailer reorder points that minimize supply chain costs given R_w. The next theorem indicates the retailers tend to choose smaller reorder points than optimal.

THEOREM 3. $\min R_r^o(R_w) \geq \underline{R}_r(R_w)$ *and* $\max R_r^o(R_w) \geq \overline{R}_r(R_w)$.

Suppose $R_r^o(R_w^o)$ is unique and (R_w^o, R_r^o) is not a Nash equilibrium. (Those conditions generally hold for the cases included in the numerical study.) Then Theorem 3 implies that in the SCI game the retailers' choose a reorder point that is lower than the optimal reorder point, even if the supplier chooses the optimal reorder point. In that case, the retailers' average inventory is too low relative to the optimal amount. (The retailers' average inventory is increasing in R_r.) Why do the retailers carry too little inventory? They shortchange the system because the marginal reduction in backorder costs (β_r) they receive by increasing R_r is less than the marginal reduction in backorder costs the supply chain receives, ($\beta = \beta_w + \beta_r$). In contrast, both Li (1992) and Mahajan and van Ryzin (1998) find that competing retailers carry too much inventory. In their models the retailers compete for customers and there is no endogenous supplier to influence the retailers' lead time. The retailers' carry too much inventory because each retailer treats a stockout as a lost sale, but because customers can switch among retailers, the sale might not be lost to the system. Hence, each retailer overestimates the cost of a stockout, and therefore stocks too much. (Similar results have been obtained in the R&D patent racing literature: Firms invest too much in R&D because they ignore the possibility that another firm might obtain the innovation. See Lee and Wilde 1980.)

For the supplier, there is no clear relationship between the competitive reorder point and the optimal reorder point. This occurs because the supplier's reorder point impacts both the retailers' average backorders and average inventory. To explain, note that increasing R_w decreases the retailers' average backorders. However, the marginal benefit of reducing the retailers' backorders is lower for the supplier (β_w) than it is for the supply chain ($\beta = \beta_w + \beta_r$). Hence, with respect to the incentive to lower the retailers' backorders, the supply chain chooses a higher reorder point than the supplier, $R_w < R_w^o$. On the other hand, increasing R_w also increases the retailers' average inventory. To choose the optimal reorder point, the supply chain must recognize that increasing R_w will increase the retailers' holding cost, but the supplier will ignore that impact because the supplier does not pay for the retailers' holding cost. Thus, with respect to the incentive to increase the retailers' inventory, the supply chain chooses a lower reorder point than the supplier, $R_w \geq R_w^o$. The numerical study confirms that either of these effects may prevail, i.e., the supplier may choose a reorder point that is smaller or larger than the optimal one, even if the retailers are choosing their optimal reorder point. In contrast, in a two-echelon setting with only one retailer and base stock ordering, Cachon and Zipkin (1999) demonstrate that the former is never dominated by the latter; the supplier never chooses in equilibrium a base stock policy that is larger than optimal.

The next theorem indicates that when the optimal solution is unique, it never occurs that both Nash equilibrium reorder points are larger than the optimal ones. In other words, if the firms switch from a Nash equilibrium to the optimal solution, then it never is the case that all firms lower their reorder points.

THEOREM 4. *Suppose there exists a unique pair of optimal reorder points,* (R_w^o, R_r^o). *For any Nash equilibrium,* (R_w^*, R_r^*), *either* $R_w^* \leq R_w^o$ *or* $R_r^* \leq R_r^o$ *(or both).*

In the numerical study there is a unique optimal pair of reorder points for all of the problems tested.

4.5. Nonidentical Retailers

With one-for-one ordering, the cost functions developed by Axsäter (1990) can be applied, even if the retailers have nonidentical Poisson demand rates. However, in that case it would be necessary to track each retailer's reorder point because a single reorder point will not necessarily minimize each retailer's cost function. However, the game is still supermodular, and iterative elimination of dominated strategies will still yield the minimal and maximal Nash equilibria. Of course, the elimination of dominated strategies would have to be performed for each retailer.

With batch ordering, the previous analysis requires identical demand rates and batch sizes, but some retailer heterogeneity is possible. Retailers could have different lead times or cost rates. (A change in one retailer's demand rate or batch size influences the other retailers' costs through a change in the lead time distribution, whereas changes in the other parameters do not.) When there is heterogeneity, as in the case of one-for-one ordering, it is necessary to track each retailer's reorder point (or at the very least to track a reorder point for each set of identical retailers).

Forsberg (1997) extends Axsäter (1993) to include nonidentical retailer demand rates and batch sizes. That work also demonstrates that each player's cost function is a convex combination of the one-for-one ordering cost functions. This suggests that the nonidentical retailer game with batch ordering is also supermodular.

5. COOPERATION

When the optimal solution is not a Nash equilibrium, at least one of the firms has an incentive to deviate from it, and by definition, this deviation will never lower supply chain costs. Given that competition can lead the supply chain to perform less efficiently than possible, how can the firms cooperate to improve their performance? This research offers three strategies: change incentives, change equilibria and change control.

5.1. Change Incentives

Suppose an optimal pair of reorder points, (R_w^o, R_r^o), is not a Nash equilibrium. The goal of changing the players incentives is to make (R_w^o, R_r^o) a Nash equilibrium once the players account for their modified incentives.

If (R_w^o, R_r^o) is to be a Nash equilibrium, then the retailers must wish to choose R_r^o, given that the supplier will choose R_w^o. From Theorem 3, the retailers need an incentive to raise their reorder point. A simple solution is to charge each an additional β_w per retailer backorder, to be paid to the supplier. With this scheme the retailers become responsible for all consumer backorder costs. Given that additional penalty, the retailers' backorder and holding cost rates equal the supply chain's cost rates, so R_r^o will minimize their costs by definition. However, that transfer payment fully compensates the supplier for the supplier's backorder costs, so now the supplier's optimal choice is to carry zero inventory, $R_w = -Q_w$. If $R_w^o > -Q_w$, another incentive is needed to get the supplier to raise its reorder point.

A supplier holding cost subsidy will not work to induce the supplier to raise its reorder point because the supplier faces no penalty for retailer backorders. However, a fee based on the *supplier's* backorders (retailer orders that have not been shipped) will induce the supplier to raise its reorder point. Specifically, the retailers can charge the supplier p_w per unit of the supplier's backorders per unit time. (Note that p_w is charged per unit and not per batch, even though the supplier's backorders always equals an integer batch quantity. The equivalent per batch penalty is $Q_r p_w$ because there are Q_r units per batch.) As p_w increases, the supplier's optimal reorder point will clearly increase. In fact, it is possible to determine upper and lower limits on the backorder penalty, $\bar{p}_w$ and $\underline{p}_w$, such that R_w^o is the supplier's optimal reorder point when $\underline{p}_w \leqslant P_w \leqslant \bar{p}_w$.

Define $I_w(R_w)$ and $B_w(R_w)$ as the supplier's expected inventory and backorders in *units* when it chooses R_w as its reorder point. The supplier will choose R_w^o to minimize its cost function, $h_w I_w(R_w) + p_w B_w(R_w)$, which is convex in R_w. In steady state, the supplier's inventory position equals its inventory minus its backorders plus on-order inventory, so

$$Q_r\left(R_w + \frac{Q_w+1}{2}\right) = I_w(R_w) - B_w(R_w) + N\lambda_r L_w. \quad (5)$$

The left-hand side above is the average inventory position in units; in steady state the supplier's inventory position (in subbatches) is uniformly distributed on the interval

$[R_w+1, R_w+Q_w]$. The last term is the expected on-order inventory in units from Little's Law.

If R_w^o minimizes the supplier's costs, then increasing the reorder point to R_w^o+1 would increase holding costs at least as much as the decrease in backorder costs for all $p_w \le \bar{p}_w$. Hence,

$$\bar{p}_w[B_w(R_w^o)-B_w(R_w^o+1)] = h_w[I_w(R_w^o+1)-I_w(R_w^o)].$$

Using Equation (5), the above simplifies to

$$\bar{p}_w = h_w \frac{I_w(R_w^o+1)-I_w(R_w^o)}{Q_r-(I_w(R_w^o+1)-I_w(R_w^o))}.$$

Similarly, the backorder penalty must be sufficiently high so that the supplier cannot reduce costs by choosing R_w^o-1,

$$h_w[I_w(R_w^o)-I_w(R_w^o-1)] = \underline{p}_w[B_w(R_w^o-1)-B_w(R_w^o)],$$

which simplifies to

$$\underline{p}_w = h_w \frac{I_w(R_w^o)-I_w(R_w^o-1)}{Q_r-(I_w(R_w^o)-I_w(R_w^o-1))}.$$

The results from Axsäter (1993) are used to evaluate $I_w(R_w)$. See Appendix B for details. Note that the retailers are unable to influence the transfer payments they receive from the supplier, since their reorder points have no impact on the supplier's backorders. Hence, assuming R_w^o is chosen, R_r^o is the retailers' optimal response no matter what p_w is chosen.

To summarize, a coordinating contract transfers from the retailers to the supplier β_w per unit of retail backorders per unit time and transfers from the supplier to the retailers $p_w \in [\underline{p}_w, \bar{p}_w]$ per unit of supplier backorders per unit time. With this contract (R_w^0, R_r^o) is a Nash equilibrium, hence supply chain costs are minimized.

Although supply chain costs will be lower by coordinating on the optimal reorder points, there is no guarantee that both players will be better off with the coordination solution instead of the competitive solution. In those cases the players can also agree to fixed transfer payments to compensate those players who experience a net cost increase. Because total supply chain costs decline by adopting the optimal reorder points, it is always possible to find a set of fixed transfer payments such that no player is worse off than with the competitive solution (once all payments are counted).

5.2. Change Equilibrium

Changing incentives is nontrivial for firms. They must change their accounting systems and establish procedures to verify compliance. They must negotiate terms. When there are multiple Nash equilibria, there may be a simpler way to improve system performance. Because supply chain costs are not necessarily identical across Nash equilibria, the firms could ensure that they coordinate on the lowest cost Nash equilibrium. Of course, this does not guarantee optimal supply chain performance unless the optimal policy is a Nash equilibrium (in which case this is a particularly effective coordination strategy). Furthermore, fixed transfer payments may still be needed to gain the acceptance of all of the players.

5.3. Change Control

The decentralized system performs poorly, in part because no single firm has control over all of the operating decisions. If a single firm is assigned control over all the supply chain's decisions, then that firm could choose optimal policies. In fact, shifting control to the supplier (vendor) is a key aspect of most VMI programs. For example, Campbell Soup established a VMI program in which Campbell Soup assumed responsibility for replenishing its retailers' inventories, even though the retailers continued to own the inventory at their locations. (See Cachon and Fisher 1997 for additional details.)

In the context of this model, shifting control to the supplier (which for simplicity will be referred to as VMI) means that the supplier chooses the retailers' reorder points in addition to its own reorder point. (Assume the supplier manages all of the retailers' inventory. Because there is no interaction between the retailers, the case of partial VMI adoption is easily handled.) VMI does not change the cost/ownership structure of the supply chain; in particular, the retailers incur holding costs for their inventory even though the supplier controls the amount of inventory they hold. Furthermore, information systems are generally required to implement VMI because the supplier must observe the retailers' inventory positions to execute their reorder point policy. In this model, it is assumed that those information systems are feasible and available; the focus is on how shifting control influences coordination and not on the many other dimensions of VMI implementation.

Even though with VMI the retailers transfer control of their inventory management to the supplier, that control is not total. If it were total, the supplier would choose $R_w = -Q_w$ and a very large R_r; the supplier would carry no inventory and there would be no consumer backorders. The supplier would have zero cost, and the retailers' costs would probably increase substantially. This does not occur because there is either an implicit or explicit understanding that the supplier can choose the reorder points as long as the retailers are no worse off than they would be under the Nash equilibrium. The "no worse off" constraint means that the retailers' total costs are not higher, accounting for transfer payments they receive. In fact, the retailers' bargaining power might be even stronger. They may insist that the ratio of their savings to the supply chain's savings be at least γ, again accounting for transfer payments. Even though the retailers impose that constraint, the supplier does have some additional flexibility when choosing reorder points: In the decentralized game, the firms choose reorder points that are in their reaction correspondences; but with VMI, the supplier is free to choose a reorder point that is not optimal for the retailers, i.e., the supplier can choose $R_r \notin R_r(R_w)$.

In this setting the supplier's VMI problem is

$$\min_{R_w, R_r, F} C_w(R_w, R_r) + NF$$
$$\text{s.t. } C_r(R_w, R_r) - F \le C_r^* - \frac{\gamma}{N}(N(C_r^* - C_r(R_w, R_r)) + C_w^* - C_w(R_w, R_r)), \quad (6)$$
$$C_w(R_w, R_r) + NF \le C_w^*$$

where C_r^* and C_w^* are a retailer's and the supplier's expected cost in the Nash equilibrium that would prevail if VMI were not implemented, and F is a fixed transfer payment per unit time per retailer. The first constraint states that a retailer's cost with VMI implemented must be no greater than its Nash equilibrium cost minus the retailer's demanded share of any realized cost savings. The second constraint requires that the supplier is also no worse off implementing VMI than it would be in the Nash equilibrium.

There is no sign restriction imposed on F. The supplier can charge the retailers for participating in VMI as long as the fee does not leave the retailers worse off. Further, this fee does not have to be an explicit cash payment. For example, the retailer could agree to accept a broader product line or the supplier could provide additional advertising support. The key feature is that the transfer payment is independent of the players' actions.

For any chosen (R_w, R_r) the supplier will minimize F, so the supplier chooses

$$F = C_r(R_w, R_r) - C_r^* + \frac{\gamma}{N}(N(C_r^* - C_r(R_w, R_r)) + C_w^* - C_w(R_w, R_r)).$$

Substituting the above into Equation (6), the supplier's problem becomes

$$\min_{R_w, R_r, F} (1-\gamma)(C_w(R_w, R_r) + NC_r(R_w, R_r)) + \gamma C_w^* - (1-\gamma)NC_r^*$$
$$\text{s.t. } (1-\gamma)(C_w(R_w, R_r) + NC_r(R_w, R_r)) \le (1-\gamma)(C_w^* + NC_r^*).$$

The final two terms in the objective function are constants. The first term is $(1-\gamma)$ times total supply chain costs, $C_w(R_w, R_r) + NC_r(R_w, R_r)$. Therefore, (R_w^o, R_r^o) minimizes the supplier's objective function. (Under this arrangement, each firm incurs a fixed fraction of the supply chain's optimal cost; hence, it is the cost equivalent of a profit sharing agreement.) That solution is feasible if the constraint is not violated,

$$(1-\gamma)(C_w(R_w^o, R_r^o) + NC_r(R_w^o, R_r^o)) \le (1-\gamma)(C_w^* + NC_r^*),$$

which holds for any $\gamma \in [0, 1]$. Therefore, as long as the firms are willing to share the benefits of VMI and they are willing to accept fixed transfer payments (in either direction), all firms can be better off with VMI, and VMI coordinates the supply chain, i.e., the supplier will choose the optimal reorder points.

It is important that firms are willing to accept fixed transfer payments. For example, a retailer refusal to pay any fixed payment could be modeled by including $F \geqslant 0$ as an additional constraint Equation (6). In that case the supplier might not choose the supply chain optimal reorder points if $C_r(R_w^*, R_r^*) > C_r(R_w^o, R_r^o)$, because then the supplier might not be able to share in some of their gains. (Narayanan and Raman 1997 assume $F \ge 0$ and observe that VMI does not always coordinate their system.) The effectiveness of VMI is also reduced if the supplier refuses to pay a transfer payment, i.e., imposing a $F \le 0$ constraint might mean that it is no longer in the supplier's interest to choose (R_w^o, R_r^o). That situation resembles a franchise fee. It is well known that franchise fees can be effective for coordinating supply chains in which only one operating decision is required for coordinating the supply chain. Franchise fees do not guarantee coordination in this supply chain because there are two operating decisions (the retailers' reorder point and the supplier's reorder point).

If all firms refuse to pay a transfer payment ($F = 0$ is required), then it is clear that the effectiveness of VMI will be quite limited. In fact, in the numerical study it was found in all scenarios that VMI provided no improvement in supply chain costs when fixed payments were forbidden.

These results provide a simple yet powerful message. Managers can first negotiate an allocation of the gains from an innovation and then leave one member of the supply chain in charge of its entire operation. With the allocation of rewards from innovation fixed *a priori*, the operating manager is left with the problem of maximizing those rewards, i.e., minimizing total supply chain cost. Interestingly, a similar arrangement has already been implemented in practice. The Duke University Medical Center (DUMC) outsourced all its inventory management to Baxter International (now Allegiance Healthcare), with the agreement that any savings would be divided by the two parties at a prespecified percentage. In that program, DUMC owned the inventory in the hospital (which was stored throughout the hospital in numerous closets, cabinets and storage rooms) just as the retailers do in this model, but Baxter determined replenishments based on usage data that Baxter employees collected daily. (Collecting those data required the manual process of visiting each stocking location and conducting a physical count via scanners.) The program was quite successful (see Bonneau et al. 1995). After its first year of operation, DUMC estimated it saved \$6.2 million in costs: \$2.5 million in inventory savings, \$2 million in labor costs and \$1.7 million from the reallocation of 15,000 square feet of space from inventory storage to other uses. In that first year, DUMC purchased \$23 million in suppliers from Baxter, but by the third year, DUMC increased its annual purchase volume to \$35 million.

6. NUMERICAL STUDY

The results in the previous sections do not indicate whether the optimal reorder points are a Nash equilibrium or whether there exists a unique Nash equilibrium. A better understanding of the relationship between the Nash equilibria and the optimal solution requires a numerical study.

Svoronos and Zipkin (1988) and Axsäter (1993) studied the 32 problems formed by all combinations of the following parameters: $Q_r = \{1, 4\}$; $Q_w = \{1, 4\}$; $N = \{4, 32\}$; $\lambda_r = \{0.1, 1\}$; $\beta = \{5, 20\}$; $L_r = L_w = \{1\}$; $h_r = h_w = \{1\}$. Table 1 reports optimal reorder point policies and their expected supply chain costs. (Axsäter 1993 also report these data.) To study Nash equilibria in these problems, it is necessary to split the backorder penalty among the supplier and the retailers. For each of the problems, $\beta_r = \alpha\beta$ and $\beta_w = (1-\alpha)\beta$, where the following splits are considered, $\alpha = \{0, 0.1, 0.25, 0.5, 0.75, 0.90, 1\}$.

Table 2 reports the number of Nash equilibria for each of the 32 problems and each α. In most scenarios (problem/α combination) there is a unique equilibrium, but in about 12% of them there are multiple equilibria; in two scenarios there are three equilibria. All the scenarios with multiple equilibria occur when the retailers' share of backorder costs is no less than 50% ($\alpha \geqslant 0.5$). This may occur because the retailer's reaction correspondence, $R_r(R_w)$, is less sensitive to the supplier's reorder point as the retailer's share of backorder costs decline. In terms of the graph in Figure 1, $R_r(R_w)$ becomes more vertical. For instance, when the retailer incurs no backorder costs, $(\alpha = 0)$, $R_r(R_w) = -Q_r$. Again, in terms of the graph in Figure 1, for there to be multiple equilibria the retailers' reaction correspondence needs a sufficient amount of "curve" to intersect with the supplier's reaction correspondence at multiple reorder points.

To compare equilibria to the optimal solution, define the *competition penalty* as the difference in supply chain costs in a Nash equilibrium over the optimal policy, measured as a percentage of optimal costs. For all scenarios, Table 3 reports the competition penalty assuming the firms choose the lowest cost Nash equilibrium. In approximately 20% of the scenarios the optimal reorder points are a Nash equilibrium, so the competition penalty is 0%. Also, the competition penalty is relatively small when the backorder penalty is split evenly between the supplier and the retailers $(\beta_w = \beta_r)$. However, the competition penalty can be quite significant when the backorder costs are uneven, in particular when the retailer's backorder penalty is low. When the

Table 1. Parameter values and the optimal policies.

Problem Number	λ_r	N	β	Q_r	Q_w	R_w^o	R_r^o	Optimal Cost
1	0.1	4	20	1	1	−1	0	4.77
2	0.1	4	20	1	4	−1	0	5.45
3	0.1	4	20	4	1	−1	0	9.62
4	0.1	4	20	4	4	−1	−1	14.03
5	0.1	4	5	1	1	0	−1	3.02
6	0.1	4	5	1	4	−1	−1	3.82
7	0.1	4	5	4	1	−1	−1	6.52
8	0.1	4	5	4	4	−2	−1	11.03
9	0.1	32	20	1	1	3	0	33.80
10	0.1	32	20	1	4	1	0	34.10
11	0.1	32	20	4	1	1	−1	68.53
12	0.1	32	20	4	4	0	−1	71.08
13	0.1	32	5	1	1	4	−1	18.85
14	0.1	32	5	1	4	3	−1	19.30
15	0.1	32	5	4	1	−1	−1	52.16
16	0.1	32	5	4	4	−2	−1	53.89
17	1	4	20	1	1	4	2	12.02
18	1	4	20	1	4	2	2	12.38
19	1	4	20	4	1	0	2	16.11
20	1	4	20	4	4	−1	2	18.57
21	1	4	5	1	1	3	1	8.13
22	1	4	5	1	4	2	1	8.44
23	1	4	5	4	1	−1	1	11.14
24	1	4	5	4	4	−2	1	13.84
25	1	32	20	1	1	32	2	84.42
26	1	32	20	1	4	30	2	84.51
27	1	32	20	4	1	8	1	111.70
28	1	32	20	4	4	6	1	112.74
29	1	32	5	1	1	30	1	55.95
30	1	32	5	1	4	29	1	56.03
31	1	32	5	4	1	7	0	78.72
32	1	32	5	4	4	5	0	79.45

Table 2. Number of Nash equilibria.

Problem	Retailer Share of Total Backorder Costs						
Number	0%	10%	25%	50%	75%	90%	100%
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1
4	1	1	1	2	3	1	1
5	1	1	1	1	1	1	1
6	1	1	1	2	2	1	1
7	1	1	1	1	1	1	1
8	1	1	1	1	2	1	1
9	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1
12	1	1	1	1	2	2	1
13	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1
15	1	1	1	2	1	1	1
16	1	1	1	2	1	1	1
17	1	1	1	1	2	2	1
18	1	1	1	1	2	1	1
19	1	1	1	1	2	1	1
20	1	1	1	1	3	1	1
21	1	1	1	1	1	1	1
22	1	1	1	1	1	1	1
23	1	1	1	1	2	1	1
24	1	1	1	1	2	1	1
25	1	1	1	2	1	1	2
26	1	1	1	2	1	1	2
27	1	1	1	1	1	2	1
28	1	1	1	1	1	1	1
29	1	1	1	2	1	1	2
30	1	1	1	2	1	1	2
31	1	1	1	1	1	1	1
32	1	1	1	2	1	1	1

Table 3. Competition penalty: difference in supply chain cost of the Nash equilibrium with the lowest total cost over the optimal cost, measured as a percentage of optimal cost.

Problem Number	Optimal Cost	Retailer Share of Total Backorder Costs						
		0%	10%	25%	50%	75%	90%	100%
1	4.77	105.0	105.0	111.3	0.0	0.0	0.0	0.0
2	5.45	93.0	93.0	93.0	4.4	4.4	36.3	36.3
3	9.62	1271.2	218.8	2.1	2.1	2.1	2.1	0.0
4	14.03	867.8	140.1	0.0	0.0	15.9	33.7	33.7
5	3.02	0.0	0.0	0.0	0.0	32.5	32.5	20.9
6	3.82	0.0	0.0	0.0	0.0	9.9	29.1	29.1
7	6.52	421.5	190.0	49.1	0.0	0.0	0.0	0.0
8	11.03	246.5	113.2	38.7	0.0	12.6	22.7	22.7
9	33.80	102.0	102.0	102.0	0.1	0.1	2.1	13.0
10	34.10	101.6	101.6	101.6	0.0	1.3	4.5	25.9
11	68.53	1407.9	244.6	0.0	0.8	0.8	14.6	12.3
12	71.08	1357.8	217.3	0.0	1.2	1.2	12.0	17.6
13	18.85	0.0	0.0	0.0	1.7	9.0	23.4	54.9
14	19.30	0.0	0.0	0.8	0.8	6.0	16.9	52.7
15	52.16	402.5	177.6	43.0	0.0	0.0	0.0	0.0
16	53.89	391.1	173.6	42.9	0.0	2.7	2.7	9.0
17	12.02	604.7	192.2	33.5	0.7	0.7	4.8	15.6
18	12.38	588.3	187.3	33.0	0.0	3.7	6.3	17.5
19	16.11	1193.5	148.3	36.3	4.6	4.0	0.6	0.6
20	18.57	1037.7	136.4	35.2	6.5	6.5	16.0	22.4
21	8.13	185.0	185.0	41.9	0.0	3.8	4.8	13.5
22	8.44	179.5	179.5	40.6	1.4	9.1	4.1	16.9
22	11.14	396.9	114.3	37.5	3.5	0.0	0.0	0.0
24	13.84	321.1	92.2	29.0	0.8	0.0	7.7	14.7
25	84.42	672.7	206.1	30.8	0.0	0.7	3.4	31.6
26	84.51	672.2	206.0	30.8	0.0	1.0	2.8	31.6
27	111.70	1351.6	155.6	31.6	0.2	2.3	4.9	16.1
28	112.74	1339.7	154.6	31.5	0.0	1.3	4.8	17.7
19	55.95	201.6	201.6	39.3	0.0	0.7	3.8	30.4
30	56.03	201.4	201.4	39.3	0.1	1.1	4.5	32.7
31	78.72	427.5	108.4	25.6	27.2	0.2	5.7	13.2
32	79.45	424.2	108.4	25.9	0.0	1.1	3.7	19.1
Minimum		0.0	0.0	0.0	0.0	0.0	0.0	0.0
Average		517.7	138.6	35.2	1.8	4.2	9.7	19.4
Median		399.7	151.4	33.3	0.0	1.3	4.8	17.6
Maximum		1407.9	224.6	111.3	27.2	32.5	36.3	54.9

retailers backorder costs ($\alpha = 0$), the median competition penalty is 400%, and can be as large as 1400%. However, even in this situation the optimal solution can be a Nash equilibrium because it is possible that $R_r^o = -Q_r$ (the retailers should not carry any inventory). Similarly, when the retailers incur all of the backorder costs, the optimal solution can be a Nash equilibrium when $R_w^o = -Q_w$ (the supplier should not carry any inventory).

There is intuition to explain why the competition penalty is generally worse for low values of α (the retailer cares little about backorders) than for high values of α (the supplier cares little about backorders). In these problems the backorder rate ($\beta = \{5, 20\}$) is much higher than the holding cost rate ($h_r = 1$). When the supplier does not care about backorders, the supplier does not carry inventory and so the retailers must cope with a long lead time. But even with a long lead time, they are able to carry a sufficient level of inventory to prevent backorders. However, when the retailers do not care about backorders, they choose to carry little inventory, and there is nothing the supplier can do to prevent backorders. When $R_r = -Q_r$, no level of supplier inventory can prevent retailer backorders, so costs increase enormously.

Cachon and Zipkin (1999) also found that supply chain efficiency does not suffer dramatically when backorder costs are evenly split, and the competition penalty is most severe when the retailer's share of backorder costs is low. But in their setting the optimal solution is never a Nash equilibrium.

Table 4 reports a comparison of supply chain inventory in the (lowest cost) Nash equilibrium relative to the optimal solution. In most scenarios, inventory is substantially lower in the Nash equilibrium than in the optimal solution. However, there are scenarios in which the Nash equilibrium inventory is substantially higher than optimal. Those scenarios occur when the supplier's backorder cost is low, so the supplier carries very little inventory. Low warehouse inventory results in long retailer lead times, which forces

Table 4. Supply chain inventory in the Nash equilibrium with the lowest total cost minus the optimal solution inventory (as a % of optimal inventory).

Problem Number	Optimal Inventory	Retailer Share of Total Backorder Costs 0%	10%	25%	50%	75%	90%	100%
1	3.27	−51	−51	−80	0	0	0	0
2	4.74	−55	−55	−55	−20	−20	22	22
3	9.22	−61	−30	−41	−41	−41	−41	0
4	11.33	−50	−25	0	0	1	36	36
5	0.67	0	0	0	0	−100	−100	389
6	1.22	0	0	0	0	142	84	84
7	5.42	−100	−85	−52	0	0	0	0
8	7.84	−27	−16	−33	0	3	5	5
9	29.80	−90	−90	−90	−3	−3	−6	−12
10	29.34	−89	−89	−89	0	−3	−6	−15
11	50.50	−90	−46	0	−7	−7	−14	46
12	52.53	−79	−44	0	−7	−7	−14	30
13	1.98	0	0	0	−40	−70	−89	1227
14	2.47	0	0	−33	−33	−60	−79	914
15	43.36	−88	−80	−45	0	0	0	0
16	45.32	−84	−76	−43	0	−7	−7	−13
17	9.14	−56	−50	−29	−10	−10	11	32
18	8.68	−48	−42	−20	0	22	11	23
19	14.10	−43	−46	−25	−27	−27	0	0
20	16.12	−38	−40	−22	−23	−23	1	1
21	4.69	−53	−53	−41	0	−17	37	4
22	5.16	−48	−48	−37	−16	−30	17	37
23	6.86	−38	−18	−38	1	0	0	0
24	8.97	−29	−13	−30	0	0	2	4
25	65.92	−85	−70	−39	−1	−4	−8	48
26	65.45	−85	−69	−37	0	−4	−7	47
27	85.32	−81	−53	−29	−4	−9	18	32
28	83.46	−78	−49	−25	0	−4	19	29
29	35.16	−84	−84	−56	0	−5	−11	13
30	35.59	−83	−83	−55	−2	−7	−13	9
31	53.12	−83	−61	−38	−43	−6	−17	3
32	51.57	−79	−63	−32	0	−6	−12	−2
Minimum		−100.0	−90.4	−90.4	−43.5	−100.0	−100.0	−14.8
Average		−58.6	−47.8	−34.8	−8.7	−9.5	−5.1	93.5
Median		−58.3	−49.9	−35.4	0.0	−6.2	0.0	10.7
Maximum		0.0	0.0	0.0	1.0	141.8	83.5	1226.5

retailers' to carry substantially more inventory than they would in the optimal solution.

Table 5 compares each player's cost in the (lowest cost) Nash equilibrium relative to their cost in the optimal solution. In none of the scenarios does the optimal solution represent a reduction in cost for all players, even though this switch may dramatically reduce total costs. Whether it is the supplier or the retailer who is worse off in the Nash equilibrium depends in part on the allocation of backorder costs. The player that incurs the majority of the backorder costs is most likely to be worse off in the Nash equilibrium relative to the optimal solution.

When there are multiple equilibria, some Nash equilibria will likely have higher total costs than others. Table 6 presents the *wrong equilibrium penalty*, which is the difference in supply chain cost of a Nash equilibrium over

Table 5. Players' costs in the lowest cost Nash equilibrium relative to their costs in the optimal solution.

Supplier	Retailer	Number of Problems — Retailer's Share of Backorder Costs 0%	10%	25%	50%	75%	90%	100%	Total
Higher	Higher	0	0	0	0	0	0	0	0
Equal or lower	Higher	0	0	1	11	25	27	27	91
Higher	Equal or lower	28	28	25	6	2	1	0	90
Equal or lower	Equal or lower	4	4	6	15	5	4	5	43

Table 6. Wrong equilibrium penalty.

Problem	Equilibrium Number	Percentage Increase in Cost of a Nash Equilibrium over Lowest Cost Equilibrium — Retailer Share of Backorder Costs 50%	75%	90%	100%
4	2	15.9	15.4	0.7	
4	3		18.9		
6	2	9.9	17.1		
8	2		8.9		
12	2		11.5		
15	2	43.0			
16	2	42.9			
17	2		4.0	2.9	
18	2		1.1		
19	2		0.6		
20	2		1.4		
20	3		9.0		
23	2		3.5		
24	2		7.7		
25	2	31.2			0.0
26	2	31.7			0.1
27	2			1.4	
29	2	39.9			1.2
30	2	40.2			1.3
32	2	26.6			

Table 7. Coordinating supplier backorder penalties.

Problem Number	λ_r	N	β	Q_r	Q_w	$\underline{p}_w$	$\bar{p}_w$
1	0.1	4	20	1	1	0.00	2.03
2	0.1	4	20	1	4	1.86	9.00
3	0.1	4	20	4	1	0.00	2.35
4	0.1	4	20	4	4	0.66	9.75
5	0.1	4	5	1	1	2.03	15.25
6	0.1	4	5	1	4	1.86	9.00
7	0.1	4	5	4	1	0.00	2.35
8	0.1	4	5	4	4	0.23	0.66
9	0.1	32	20	1	1	1.52	3.56
10	0.1	32	20	1	4	0.94	1.98
11	0.1	32	20	4	1	1.06	5.28
12	0.1	32	20	4	4	1.00	3.84
13	0.1	32	5	1	1	3.56	8.49
14	0.1	32	5	1	4	4.22	9.36
15	0.1	32	5	4	1	0.00	0.20
16	0.1	32	5	4	4	0.11	0.31
17	1	4	20	1	1	1.69	3.65
18	1	4	20	1	4	1.09	2.17
19	1	4	20	4	1	0.12	0.71
20	1	4	20	4	4	0.25	0.75
21	1	4	5	1	1	0.77	1.69
22	1	4	5	1	4	1.09	2.17
23	1	4	5	4	1	0.00	0.12
24	1	4	5	4	4	0.09	0.25
25	1	32	20	1	1	1.21	1.60
26	1	32	20	1	4	1.05	1.38
27	1	32	20	4	1	0.37	0.70
28	1	32	20	4	4	0.26	0.47
29	1	32	5	1	1	0.68	0.91
30	1	32	5	1	4	0.79	1.05
31	1	32	5	4	1	0.19	0.37
32	1	32	5	4	4	0.14	0.26

the lowest cost Nash equilibrium, measured as a percentage of the lowest cost Nash equilibrium cost. When the backorder penalty is evenly split ($\alpha = 0.5$), the results from Table 3 and 6 indicate that competition does not necessarily deteriorate supply chain efficiency too much, but choosing the wrong equilibrium will increase cost significantly (on average about 30%). However, the penalty for choosing the wrong equilibrium is less significant when the backorder penalty is unevenly divided, probably because there is already a high competition penalty for the lowest cost Nash equilibrium.

Table 7 reports data on supply chain coordinating contracts. Recall that in that contract, the retailers are charged an additional β_w per backorder per unit time (a fee paid to the supplier) and the supplier is charged $ap_w \in [\underline{p}_w, \bar{p}_w]$ per unit time backorder penalty for each unit the supplier backorders. In all situations, $\underline{p}_w < \beta$, which means that the supplier can be charged less for its backorders than actual consumer backorder costs. When $\underline{p}_w = 0$, $R_w^o = -Q_w$, so there is no need to charge the supplier a backorder penalty cost to raise its reorder point. There exists four scenarios in which $\bar{p}_w > \beta$, meaning that the supplier is charged even more for its backorders than the supply chain's actual backorder cost. Those scenarios occur when the retailers are not restricted to order in large quantities, $Q_r = 1$, demand is slow, $\lambda_r = 0.1$, and retail backorder costs are not too high, $\beta = 5$.

7. CONCLUSION

This research investigates competitive behavior in the supply chain inventory game. There is one supplier and N retailers. Retailers incur holding and backorder costs. The supplier incurs holding costs and backorder penalty costs and backorders at the retail level, reflecting the supplier's desire to maintain availability of its product to consumers. Using the theory of supermodular games, it is shown that Nash equilibria exist in reorder point policies. From a numerical study, it is found that the supply chain optimal reorder points are frequently not a Nash equilibrium. In these cases, the firms could agree to choose the optimal reorder points, but at least one of the firms has a private incentive to deviate. When the players incur backorder costs equally, costs in the lowest cost Nash equilibrium are generally not substantially higher than costs with the optimal reorder points. In these scenarios competition does not degrade supply chain efficiency dramatically. However, when players have divergent preferences toward consumer backorders, competition can degrade supply chain efficiency enormously, in particular when most of the backorder costs are allocated to the supplier. Hence, with inventory management, the benefit of supply chain cooperation is context specific.

Several cooperation strategies are available to the firms to help improve supply chain performance: change incentives, change equilibrium, or change control. A supply chain coordinating contract is presented that changes the players incentives, so that the optimal policy is a Nash equilibrium. This contract imposes an additional penalty on retailers for retail backorders and also imposes a penalty on the supplier for its backorders (retail orders that have not been shipped). When there are multiple Nash equilibria, it is shown that switching to the lowest cost equilibrium can reduce costs substantially, but this action does not guarantee supply chain performance. Finally, the firms could let the supplier control the supply chain's reorder point policies, subject to the constraints that all players are no worse off and any potential savings are shared. With this VMI arrangement, it is shown that the supplier will indeed choose the optimal supply chain reorder point policies.

APPENDIX A. SUMMARY OF MAJOR NOTATION

N:	number of retailers
λ_r:	mean Poisson demand rate at each retailer
Q_r:	retailer order quantity multiple, in units
Q_w:	supplier order quantity multiple, in subbatches, where a subbatch is Q_r units
L_r:	shipment time between the supplier and a retailer
L_w:	shipment time between the supplier's source and the supplier
R_r:	a retailer's reorder point, in units
R_w:	the supplier's reorder point, in batches
h_r:	a retailer's holding cost rate per unit
h_w:	the supplier's holding cost rate per unit
β_r:	a retailer's backorder cost rate per unit
β_w:	the supplier's backorder cost rate per unit (based on retailer backorders)
σ_r, σ_w:	the players' strategy spaces, $\sigma_r \in [-Q_r, \widehat{R}]$, $\sigma_w \in [-Q_w, \widehat{R}]$, where $\widehat{R}$ is a very large integer constant
$c(R_w, R_r)$:	expected supply chain cost per unit time when $Q_r = 1$ and $Q_w = 1$
$c_r(R_w, R_r)$:	a retailer's expected cost per unit time when $Q_r = 1$ and $Q_w = 1$
$c_w(R_w, R_r)$	the supplier's expected cost per unit time when $Q_r = 1$ and $Q_w = 1$
$C(R_w, R_r)$:	expected supply chain cost per unit time
$C_r(R_w, R_r)$:	a retailer's expected cost per unit time
$C_w(R_w, R_r)$:	the supplier's expected cost per unit time
$R_w(R_r)$:	the supplier's reaction correspondence, $R_w(R_r) = \arg\min_x C_w(x, R_r)$
$R_r(R_w)$:	a retailer's reaction correspondence, $R_r(R_w) = \arg\min_x C_r(R_w, x)$
R_w^o, R_r^o:	supply chain optimal reorder points
$R_w^*(R_r^*)$:	Nash equilibrium reorder points

APPENDIX B. EVALUATING COSTS AND SUPPLIER INVENTORY

Axsäter (1990) assumes $Q_w = 1$ and $Q_r = 1$, hence the firms implement base stock policies. He defines S_w as the supplier's order-up-to level, so the supplier's reorder point is $S_w - 1$. The retailers' order-up-to level is S_r, so their reorder point is $S_r - 1$. (If retailers have nonidentical demand rates, then their base stock policies may differ.) Axsäter (1990) defines $\Pi^{S_r}(S_w)$ as the expected holding and backorder costs per unit of demand at a retailer, where holding costs are incurred at rate h_r per unit and backorder costs are incurred at rate β per unit. Also, Axsäter (1990) defines $\gamma(S_w)$ as the expected holding costs incurred by supplier per unit demand. Therefore,

$$c(R_w, R_r) = N\lambda_r(\Pi^{R_r+1}(R_w+1) + \gamma(R_w+1)).$$

To evaluate $c_r(R_w, R_r)$, follow the evaluation of $c(R_w, R_r)$ but set the supplier's holding cost rate to zero ($h_w = 0$) and the retailer's backorder cost rate to β_r, then divide $c(R_w, R_r)$ by N. To evaluate $c_w(R_w, R_r)$, follow the evaluation of $c(R_w, R_r)$ but set the retailer's holding cost rate of zero ($h_r = 0$) and the retailer's backorder cost rate to β_w.

Since $N\lambda_r\gamma(R_w+1)$ is the supplier's expected inventory costs,

$$I_w(R_w) = N\lambda_r\gamma(R_w+1)/h_w.$$

APPENDIX C. PROOFS

PROOF OF LEMMA 1. From (29) in Axsäter (1990),

$$\prod^{S_r-1}(S_w) - \prod^{S_r}(S_w) \leq \prod^{S_r-1}(S_w-1) - \prod^{S_r}(S_w-1). \qquad \text{(C-1)}$$

$\prod^{S_r}(S_w)$ and $\prod_r^{S_r}(S_w)$ differ only in the retailer backorder rates (i.e., β and β_r, respectively),

$$\prod_r^{S_r-1}(S_w) - \prod_r^{S_r}(S_w) \leq \prod_r^{S_r-1}(S_w-1) - \prod_r^{S_r}(S_w-1).$$

Because $c_r(R_w, R_r) = \lambda_r \prod_r^{R_r+1}(R_w+1)$, the above can be written as

$$c_r(R_w+1, R_r) - c_r(R_w+1, R_r+1) \\ \leq c_r(R_w, R_r) - c_r(R_w, R_r+1),$$

thereby confirming decreasing differences for the retailer. The approach for the supplier is almost identical. $\prod^{S_r}(S_w)$ and $\prod_w^{S_r}(S_w)$ differ only in the backorder rates (β and β_w, respectively) and holding cost rates (h_r and 0, respectively). So from Equation (C-1),

$$\prod_w^{S_r-1}(S_w) - \prod_w^{S_r-1}(S_w-1) \leq \prod_w^{S_r}(S_w) - \prod_w^{S_r}(S_w-1).$$

Because $c_w(R_w, R_r) = \lambda_w(\prod_w^{R_r+1}(R_w+1) + \gamma(R_w+1))$, the above can be written as

$$c_w(R_w+1, R_r) - c_w(R_w, R_r) \\ \leqslant c_w(R_w, +1, R_r+1) - c_w(R_w, R_r+1),$$

which demonstrates decreasing differences. □

PROOF OF LEMMA 2. For notational convenience, define $\psi_r^k(i)=C_r(i,k+1)-C_r(i,k)$. From (2), for $\delta\in\{0,1\}$,

$$C_r(R_w+\delta,R_r+1)-C_r(R_w+\delta,R_r)$$
$$=\Big(\frac{1}{Q_wQ_r}\Big)\Bigg[\sum_{j=\max\{1,-R_w-\delta-Q_w\}}^{-R_w-\delta-1}\sum_{k=R_r+1}^{R_r+Q_r}\sum_{m=0}^{jQ_r}q_{m,j}\psi_r^{k-m-1}(-1)+\sum_{j=\max\{0,R_w+\delta+1\}}^{R_w+\delta+Q_w}\sum_{k=R_r+1}^{R_r+Q_r}\sum_{i=I_j^l}^{I_j^u}p_{i,j}\psi_r^{k-1}(i-1)\Bigg].$$

Two cases are considered: $R_w<-1$, and $R_w\geq -1$.

Begin with $R_w<-1$. In this case, Equation (4) can be written as

$$\sum_{j=1}^{-R_w-2}\sum_{k=R_r+1}^{R_r+Q_r}\sum_{m=0}^{jQ_r}q_{m,j}\psi_r^{k-m-1}(-1)+\sum_{j=0}^{R_w+1+Q_w}\sum_{k=R_r+1}^{R_r+Q_r}\sum_{i=I_j^l}^{I_j^u}p_{i,j}\psi_r^{k-1}(i-1)$$
$$\geq\sum_{j=1}^{-R_w-1}\sum_{k=R_r+1}^{R_r+Q_r}\sum_{m=0}^{jQ_r}q_{m,j}\psi_r^{k-m-1}(-1)+\sum_{j=0}^{R_w+Q_w}\sum_{k=R_r+1}^{R_r+Q_r}\sum_{i=I_j^l}^{I_j^u}p_{i,j}\psi_r^{k-1}(j-1),$$

which simplifies to

$$\sum_{k=R_r+1}^{R_r+Q_r}\sum_{i=I^l_{R_w+1+Q_w}}^{I^u_{R_w+1+Q_w}}p_{i,R_w+1+Q_w}\psi_r^{k-1}(i-1)\geq\sum_{k=R_r+1}^{R_r+Q_r}\sum_{m=0}^{(-R_w-1)Q_r}q_{m,-R_w-1}\psi_r^{k-m-1}(-1).\tag{C-2}$$

From Lemma 1, $c_r(R_w,R_r)$ has decreasing differences in R_r, so $\psi_r^k(i+1)\geq\psi_r^k(i)$. Furthermore, $I^l_{R_w+1+Q_w}>0$. Because $c_r(R_w,R_r)$ is convex in R_r, $\psi_r^k(i)\geq\psi_r^{k-1}(i)$. Combining those results, for $k\in[R_r+1,R_r+Q_r]$,

$$\sum_{i=I^l_{R_w+1+Q_w}}^{I^u_{R_w+1+Q_w}}p_{i,R_w+1+Q_w}\psi_r^{k-1}(i-1)\geq\psi_r^{k-1}(-1)\geq\sum_{m=0}^{(-R_w-1)Q_r}q_{m,-R_w-1}\psi_r^{k-m-1}(-1),$$

which confirms (C-2).

Now consider $R_w\geq -1$. Arrange (4) as

$$\sum_{j=R_w+1}^{R_w+Q_w}\sum_{k=R_r+1}^{R_r+Q_r}\sum_{i=I_{j+1}^l}^{I_{j+1}^u}p_{i,j+1}\psi_r^{k-1}(i-1)\geq\sum_{j=R_w+1}^{R_w+Q_w}\sum_{k=R_r+1}^{R_r+Q_r}\sum_{i=I_j^l}^{I_j^u}p_{i,j}\psi_r^{k-1}(i-1).\tag{C-3}$$

Because the first two summations on either side of the inequality are identical, the above holds if the following can be shown for each $j\in[R_w+1,R_w+Q_w]$ and $k\in[R_r+1,R_r+Q_r]$,

$$\sum_{i=I_{j+1}^l}^{I_{j+1}^u}p_{ij+1}\psi_r^{k-1}(i-1)\geq\sum_{i=I_j^l}^{I_j^u}p_{ij}\psi_r^{k-1}(i-1).\tag{C-4}$$

Because P_{j+1} stochastically dominates P_j and the summation interval in the left-hand side of (C-4) begins at a higher value and contains no fewer elements (i.e., recall that, $I_{j+1}^l>I_j^l$ and $I_{j+1}^u-I_{j+1}^l\geq I_j^u-I_j^l$), so (C-4) does in fact hold. This can be shown more explicitly. Because $\psi_r^k(i)$ is increasing in i, the following is a lower bound for the left hand side of (C-4),

$$\sum_{i=I_{j+1}^l}^{I_j^u-1}p_{i,j+1}\psi_r^{k-1}(i-1)+\psi_r^{k-1}(I_j^u-1)\sum_{i=I_j^u}^{I_{j+1}^u}p_{i,j+1},$$

which can be written as

$$\psi_r^{k-1}(I_j^u-1)-\sum_{i=I_{j+1}^l}^{I_j^u-1}\Pr(P_{j+1}\leq i)\big(\psi_r^{k-1}(i)-\psi_r^{k-1}(i-1)\big).$$

Similarly, the following is an upper bound for the right-hand side of Equation (C-4),

$$\psi_r^{k-1}(I_{j+1}^l)-\sum_{i=I_j^l}^{I_{j+1}^l}p_{i,j}+\sum_{i=I_{j+1}^l+1}^{I_j^u}p_{i,j}\psi_r^{k-1}(i-1),$$

which can be written as

$$\psi_r^{k-1}(I_j^u)-\sum_{i=I_{j+1}^l}^{I_j^u-1}\Pr(P_j\leq i)(\psi_r^{k-1}(i)-\psi_r^{k-1}(i-1)).$$

Because $\Pr(P_{j+1}\leq i)\leq\Pr(P_j\leq i)$ and $\psi_r^{k-1}(i)\geq\psi_r^{k-1}\times(i-1)$, it follows that

$$\psi_r^{k-1}(I_j^u)-\sum_{i=I_{j+1}^l}^{I_j^u-1}\Pr(P_{j+1}\leq i)(\psi_r^{k-1}(i)-\psi_r^{k-1}(i-1))\geq\psi_r^{k-1}(I_j^u)-\sum_{i=I_{j+1}^l}^{I_j^u-1}\Pr(P_j\leq i)(\psi_r^{k-1}(i)-\psi_r^{k-1}(i-1)).$$

Hence, the lower bound for the left-hand side of Equation (C-4) is at least as large as the upper bound for the right-hand side of (C-4), which implies that (C-4) indeed holds.

The same analysis applied to the supplier's cost function (i.e., merely change the "r" subscripts to "w") demonstrates that $C_w(R_w,R_r)$ has decreasing differences in R_w. □

PROOF OF THEOREM 3. From Milgrom and Roberts (1990), the SCI game is supermodular if: (1) σ_w and σ_r are complete lattices; (2) each player's payoff function is order upper semicontinuous in its reorder point (holding the other players' reorder points fixed) and order continuous in the other players' reorder points (holding its reorder point fixed); (3) the payoff functions have finite upper bounds; (4) each player's payoff function is supermodular in its reorder point; and (5) the payoff functions have increasing differences in the reorder points. In the SCI game a player's payoff function is the negative of its cost function: (1) The strategy spaces are lattices because they are single dimensional and bounded. The retailer's strategy space is complete because for all nonempty subsets $T \subset \sigma_r$, $\inf(T) \in \sigma_r$ and $\sup(T) \in \sigma_r$. (2) Order upper semicontinuity and order continuity are easy to confirm since each player has a finite number of strategies. (3) The payoff functions have an upper bound because a minimum (finite) cost exists. (4) The supplier's payoff function is supermodular in R_w if for all $R_w, R'_w \in \sigma_w$,

$$C_w(R_w, R_r) + C_w(R'_w, R_r) \leq C_w(R_w \wedge R'_w, R_r) + C_w(R_w \vee R'_w, R_r),$$

where $R_w \wedge R'_w$ is the *meet* of R_w and R'_w, and $R_w \vee R'_w$ is the *join* of R_w and R'_w. The meet of R_w and R'_w is the supremum of $\{R_w, R'_w) = \max\{R_w, R'_w)$, and the join of R_w and R'_w is the infimum of $\{R_w, R'_w)$. Hence, the above follows immediately. The result for the retailers follows analogously. (5) Increasing difference in the payoff functions is confirmed by Lemma 2, which shows that there are decreasing differences in the cost functions.

Because the SCI game is supermodular, existence of a Nash equilibrium follows from Theorem 5's first corollary in Milgrom Roberts (1990). □

PROOF OF THEOREM 4. Because the SIC game is supermodular, the result follows from Theorem 5 in Milgrom and Roberts (1990). □

PROOF OF THEOREM 5. Supplier holding costs are independent of R_r, so R_r only influences costs at the retailer level. $C(R_w, R_r)$ and $C_r(R_w, R_r)$ are convex in R_r. Holding costs in both $C(R_w, R_r)$ and $C_r(R_w, R_r)$ are charged at rate h_r, but backorder costs in $C(R_w, R_r)$ are no lower than in $C_r(R_w, R_r)$, i.e. $\beta_r \leq \beta$. Holding costs are convex and nondecreasing in R_r, while backorder costs are convex and nonincreasing in R_r, which implies that

$$C_r(R_w, R_r) - C_r(R_w, R_r + 1) \leq C(R_w, R_r) - C(R_w, R_r + 1).$$

Suppose $\overline{R}_r(R_w) > \max R_r^o(R_w)$. For $R'_r < \overline{R}_r(R_w)$, $C_r(R_w, R'_r) - C_r(R_w, R'_r + 1) \geq 0$, i.e. increasing R_r does not raise costs. But then $C(R_w, R'_r) - C(R_w, R'_r + 1) \geq 0$, which means that $\overline{R}_r(R_w) > \max R_r^o(R_w)$ cannot hold. Similar argument demonstrates $\min R_r^o(R_w) \geq \underline{R}_r(R_w)$. □

PROOF OF THEOREM 6. Let (R_w^o, R_r^o) be the unique optimal order points and let (R_w^*, R_r^*) be a Nash equilibrium. Assume $R_w^*, > R_w^o$ and R_r^*, R_r^o. From Theorem 5, $\overline{R}_r(R_w^o) \leq R_r^o$, i.e., when the supplier chooses R_w^o, the retailer is not choosing a reorder point above R_r^o. Because $C_r(R_w, R_r)$ has decreasing differences in R_r, $\overline{R}_r(R_w^o + 1) \leq \overline{R}_r(R_w^o)$, i.e., the retailer does not raise its reorder point when the supplier raises its reorder point. But $R_w^* > R_w^o$ has been assumed, so $\overline{R}_r(R_w^*) \leq \overline{R}_r(R_w^o) \leq R_r^o$. Because by definition $R_r^* \leq \overline{R}_r(R_w^*)$, it must hold that $R_r^* \leq R_r^o$, which contradicts the original assumptions. □

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