Pricing Control and Regulation on Online Service Platforms

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Abstract

An open debate in platform design is who should control pricing: the platform (centralized pricing), or its service providers (decentralized pricing)? We show a key trade-off is between regulating competition and enabling price tailoring. Centralized pricing allows the platform to manage competition but it faces information asymmetry, as it cannot observe agent costs. Decentralized pricing lets agents adjust prices to their costs, but without oversight, competition can become too strong (prices too low) or too weak (prices too high). For commission-based platforms, either form of price control can prevail depending on market conditions, implying neither dominates. However, a relatively simple tweak – adopting an affine fee structure based on posted prices or quantities served – allows the platform to decentralize pricing control without sacrificing optimality. This flexibility further supports agent classification as independent contractors, offering platforms a valuable strategic option for how to structure their workforce.

1 Introduction

Online service platforms create marketplaces to match independent agents with potential customers. They have been established in many domains, including IT (e.g., Microsoft Azure), ride sharing (e.g., Uber), food delivery (e.g., DoorDash), freelance labor (e.g., TaskRabbit), handmade and vintage products (e.g., Etsy), accommodations (e.g., AirBnB), mobile phone applications (e.g., App Store), and physical goods (e.g., Amazon, Temu), etc.

Pricing is a critical function for a platform's success, both who selects prices and how revenue is generated. On some platforms, agents post their desired price. We refer to this as *decentralized*

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pricing. On others, the platform selects the price for the agents, which we refer to as *centralized pricing*. The platform can collect revenue with a commission on the sales price, as is commonly observed, or some other, more elaborate structure. There is little guidance in the literature to choose among these options.

Although many platforms have only operated with a single pricing structure, most participate in evolving markets and different approaches have been taken in similar settings. For example, third-party sellers on Amazon set their own prices, but on Temu prices are set by the platform. Ride-sharing generally has centralized pricing, but Uber experimented with decentralized prices set by drivers. Government regulation is also a factor in this decision. For example, state and federal governments (Bhuiyan 2020, Ongweso 2021, O'Brien 2021) have passed laws explicitly recognizing the freedom to set prices as an important criterion in classifying a worker as a contractor rather than an employee of the platform.

To address the issue of price control on a platform, we consider a platform with the following characteristics: (i) a large number of independent service providers, who we refer to as *agents*, offer their services (or products), (ii) agents have private knowledge of their heterogeneous costs, (iii) agents determine their level of participation on the platform, (iv) all participants on the platform choose actions to maximize their objective, correctly anticipating the actions of the other participants, and (v) agents offer differentiated goods, so the consumer choice process depends on the set of available prices on the platform. Many agents (characteristic i) implies agents know they individually have no ability to influence the aggregate outcomes on the platform. However, they are sophisticated enough to respond to what happens on the platform in a manner that is best for them (characteristic iv). Agent costs (characteristic ii), which include both out-of-pocket explicit costs and opportunity costs, vary considerably, and cannot be inferred by the platform (i.e., even after best effort estimation) (Chen et al. (2019)). The lack of cost visibility poses a challenge for the platform to make decisions that accommodate agents' heterogeneous preferences, especially given that agents control how much they work (characteristic iii). For example, Filippas et al. (2023) empirically show that centralized pricing harms agent participation in a vehicle rental platform due to its inability to fully account for the agents' costs. Finally, decentralized control of pricing can lead to price dispersion because each agent has an opportunity to attract some demand even if the agent does not have the lowest price (characteristic v).

For the designer of the platform, the key trade-off is between the regulation of agent competition and the facilitation of agent price tailoring. When agents are allowed to compete on their own through decentralized pricing, agents' tailor their prices to their costs which leads to price dispersion.

Price dispersion can be good for the platform: low-cost agents choose lower prices, serving more customers at an attractive price, and even though high-cost agents charge more, doing so allows them to participate on the platform, which expands total supply and overall welfare. However, there is no guarantee that decentralized pricing leads to the right amount of dispersion or even the right average price. If competition is too weak, all prices are too high, making the platform relatively unattractive, lowering demand. If competition is too strong, prices are too low, commission revenue declines and some agents exit the market. Centralized pricing allows the platform to ensure the average price on the platform is to its liking. But the limitation of "one size fits all" pricing is that, by definition, it eliminates the benefits of price dispersion. On balance, decentralized pricing is preferred as long as competition is not too intense, but otherwise the platform prefers to switch to centralized pricing even though it too is not ideal.

We find that a relatively simple tweak to the fee structure – moving from a commission, which is linear in price, to one which is affine in either price, or in quantity served – suffices to overcome the limitations of decentralized pricing and achieve optimality for the platform. We refer to the first type of fee structure as *commission-plus*, and the second as *quantity-based*. More specifically, the per-unit fee the agent pays the platform has two components. The first depends either on the agent's price (commission-plus) or the agent's demand (quantity-based), and can either be a surcharge (i.e., per-unit fees that increase in price or quantity) or a discount (i.e., per-unit fees that decrease in price or quantity). The second is a base per-unit fee which is independent of the agent's actions or the market outcomes. We demonstrate that both fee structures enable the platform to indirectly regulate agent competition, while still allowing agents to set prices in response to their own costs. Not only do these fee structures improve upon decentralized pricing with just the commission fee, they are always better for the platform than centralized pricing, and most importantly, we show there does not exist another fee structure that does better (i.e., it is an optimal fee structure).

In sum, through properly designed and relatively simple fee structures, a platform can delegate control of pricing to agents while also receiving the maximum profit possible. To the best of our knowledge, we are the first paper that characterizes whether and how a platform can optimally delegate pricing decisions to a continuum of competing agents with private cost information.

2 Literature Review

This work is related to research on (i) price delegation, (ii) ownership and contracts in supply chains, and (iii) platform management.

Price Delegation

One portion of the price delegation literature specifically focuses on a principal's decision to grant pricing authority to sales agents who can engage in costly effort to increase demand and/or have private information regarding demand (e.g., Weinberg 1975, Lal 1986, Joseph 2001, Desiraju and Moorthy 1997, Bhardwaj 2001, Mishra and Prasad 2004, 2005, Atasu et al. 2021). Additional work considers a broader context for a principal's price delegation decision with agents who are independent sellers or contractors: (e.g., Foros et al. 2009, Jerath and Zhang 2010, Abhishek et al. 2016, Hagiu and Wright 2015, Johnson 2017, Foros et al. 2017, Hagiu and Wright 2019a). Much of this work includes some form of moral hazards (e.g., costly effort), some papers consider agents with private demand information and several focus a retailer's preference for the agency model (i.e., suppliers select prices) or a reselling model (i.e., suppliers set per-unit wholesale prices for their products while the retailer retains price control). The key differences with our work include (i) we consider a platform with a large number of relatively small agents, (ii) agents' costs are heterogeneous, and (iii) each agent's cost is private information. Consequently, unlike in previous work, the principal is always responsible for designing the fee structure, but is also somewhat limited in its ability to extract rents from agents (e.g., it is not optimal to merely use a fixed fee). Furthermore, the set of active agents on the platform is endogenous as well as their level of engagement. Overall, given the differences across settings, our work should be viewed as complementary to previous findings on price delegation.

Supply Chains

Service platforms share similarities with traditional supply chains. A single firm (either a platform or supplier) offers a good or service, which is distributed through many independent, self-interested, agents such as retailers or distributors. Identified early on, retailers (agents) do not necessarily choose the prices the supplier (principal) prefers (Spengler (1950)). Consequently, a supplier may attempt to regulate prices through contractual terms and/or restraints on retail business practice (e.g. Dixit 1983, Rey and Tirole 1986, Deneckere et al. 1996, Padmanabhan and Png 1997, Dana and Spier 2001, Cachon and Lariviere 2005, Song et al. 2008). Asymmetry in cost information creates an additional complication for the principal (e.g., Corbett and De Groote 2000, Ha 2001, Corbett et al. 2004, Mukhopadhyay et al. 2008, Yao et al. 2008, Xie et al. 2014, Ma et al. 2017), and there is some work that includes competing agents (e.g., Cachon and Zhang 2006), but previous work does not consider a large number of small agents.

Platforms

There is a growing literature specifically focused on the management of platforms. Some studies only consider fixed prices and focus on the matching process that occurs among the platform participants (Arnosti et al. 2021, Feng et al. 2021, Afèche et al. 2023, Hu and Zhou 2022, Özkan and Ward 2020, Halaburda et al. 2018). Others consider revenue maximizing pricing and fee structures only with a centralized pricing regime: Riquelme et al. (2015), Gurvich et al. (2019), Cachon et al. (2017), Bai et al. (2018), Taylor (2018), Hu and Zhou (2019), Benjaafar et al. (2022), Bimpikis et al. (2016), Ma et al. (2020), Besbes et al. (2021), Castillo et al. (2017), Hu et al. (2021). And there is a set of papers that only consider decentralized pricing: Allon et al. (2012), Birge et al. (2020), Ke and Zhu (2021). Related to our interest in centralized pricing, Aouad et al. (2023) study the search frictions in dynamic matching markets between agents with heterogeneous costs and customers with different values. They demonstrate that decentralized pricing lowers social welfare. They do not consider various fee structures to maximize the platform's profit.

In settings without asymmetric information, there is some work that considers a commission with a per-unit fee payment structure, but with the goal of maximizing system profit (rather than the platform's profit): Feldman et al. (2023), Cachon and Lariviere (2005). Lobel et al. (2024) demonstrate the value of classifying agents as contractors but do not consider how decentralized pricing can be used to achieve this.

Finally, there are several papers that explicitly consider competition among platforms and focus on issues other than who controls prices: Chen et al. (2023), Ahmadinejad et al. (2019), Lian et al. (2022), and Cohen and Zhang (2022).

3 Platform Model

We model a platform that mediates transactions between customers and a large population of independent agents who offer a differentiated service. All participants are rational, risk neutral, and seek to maximize their earnings. The platform's design for this market consists of (i) the fees agents pay to participate on the platform and (ii) a pricing regime: with *decentralized pricing* each agent selects their own price, whereas with *centralized pricing* the platform chooses a single price that applies to all agents on the platform. Previous work refers to these two modes as *agency selling*, which is often contrasted with a *reselling* mode in which the platform/principal selects the price (as with our centralized pricing) but the sellers/agents select the per-unit fee payment terms.

Agents

There is a unit mass of agents with heterogeneous costs: an agent incurs a cost c per unit of demand served, where c is uniformly distributed on the $[0, 1]$ interval. The distribution of costs across agents is commonly known, but each agent's own cost is private information. Let $p(c)$ be the per-unit price for the service offered by the agent with cost c. To ease exposition, when there is no ambiguity, we occasionally use the shorthand notation $p(c) = p$. With centralized pricing, p is chosen by the platform, whereas with decentralized pricing, each agent c chooses its own price, $p(c)$.

Demand

As each agent offers a differentiated service, they can potentially attract some demand even if they do not have the lowest price in the market. For example, agents could differ in their locations (which could influence the time for the agent to reach a customer or the proximity of nearby attractions), or in their skills or style (some are more suitable for a customer's needs than others). Reflecting these characteristics of the market, an agent with price p earns demand $q(p, \overline{p})$,

$$q(p, \overline{p}) = 1 - \beta\overline{p} + \gamma(\overline{p} - p), \tag{1}$$

where $\beta > 0$ and $\gamma > 0$ are exogenous constants, and $\overline{p}$ is the weighted average price in the market,

$$\overline{p} = \frac{\int_0^1 q(p(c))p(c)dc}{\int_0^1 q(p(c))dc},$$

where $q(c)$ is the equilibrium demand of an agent with cost c. For ease of exposition we refer to $\overline{p}$ merely as the "average price". Note, $\overline{p}$ is indeed the average price paid on the platform.

The demand for the agents on the platform can be interpreted to come from two components. We refer to the first, $\beta\overline{p}$, as *platform attractiveness*. Naturally, total demand on the platform decreases as the average price on the platform, $\overline{p}$, increases. The parameter β measures the strength of the platform attractiveness effect.

We refer to the second component of demand, $\gamma(\overline{p} - p)$, as *platform competitiveness*. This is the adjustment in demand an agent receives based on the relationship between the agent's price and the average price in the market, $\overline{p}$. If the agent's price matches the average price in the market, then the agent receives the base allocation of demand. But if the agent's price deviates from the average, then the agent's share increases or decreases as expected. The parameter γ measures the strength of the platform competitiveness effect.

The balance of the platform attractiveness and competitiveness effects hinges on the relative magnitudes of β and γ. When $\gamma > \beta$, each agent's demand is increasing in the platform's average price, $\overline{p}$. In these markets, consumers view the agents' services primarily as substitutes, so the demand for one agent's service increases as all other agents become more expensive. This is what typically occurs in retail markets – one seller's demand increases when the other sellers in the market charge higher prices. However, in markets with $\gamma < \beta$, each agent's demand decreases in the platform's average price, $\overline{p}$. In these markets each agent receives a relatively constant fraction of total demand, so an agent's share of the market demand can decrease when the market's average price becomes less attractive to consumers.

A commonly used demand model in the literature takes the same form as (1), but the weighted average price, $\overline{p}$, is replaced with the straight average price (Singh and Vives 1984, Cooper and John 1988, Ledvina and Sircar 2012). Like that model, (1), can be derived from a consumer choice process (Appendix A). However, the straight average price model is ill suited for markets in which the number of participating agents is endogenous, i.e., agents are allowed to exit. This is not a concern when agents have homogeneous costs (as is typically assumed), but it is a significant limitation with heterogeneous costs. Nevertheless, our qualitative results continue to hold with the straight average price model (Online Appendix C). Given that (1) accommodates markets with agent entry and exit, it may be of independent interest for future work.

Fee Structure

The platform chooses how agents pay to participate on the platform. We first consider the common fixed commission, i.e., each agent pays the platform $\phi \leq 1$ for each unit of revenue the agent collects. Commission rates can vary in practice from platform to platform but typically range from 15% on the low end (lodging) to upwards of 50% (ride-sharing). We later consider the platform's optimal fee structures.

Sequence of Events and Information

Figure 1 displays the sequence of events. The platform first establishes who controls prices and announces the fixed commission rate $\phi < 1$. Next, agents observe the platform's payment terms and their own private cost. Then, agents decide whether or not to participate on the platform, based on their expectation for the average price on the market and their resulting demand. If they participate and the platform has chosen a decentralized pricing policy, then they next select a price. Posted prices are publicly observable. The agents' value for their outside option is normalized to zero. Finally, given price and participation decisions, demand occurs, revenue is earned, and the platform collects its share of revenue according to its payment structure. All aspects of the market (e.g., sequence of events, demand parameters, etc.) are common knowledge with the sole exception of each agent's private cost.

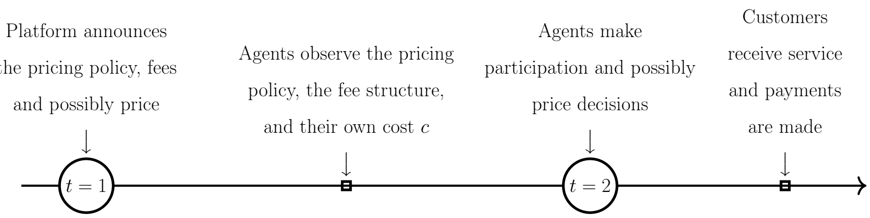

Figure 1: Sequence of events

Price Equilibrium

Agents recognize that they as small actors, a single agent has no meaningful impact on the average price, $\overline{p}$. However, the collective actions of the agents clearly do influence the average price. Given a pricing policy and fee structure, a set of prices, one for each agent type, $p(c)$, is an equilibrium if no agent has an incentive to choose a different price conditional that all other agents select the equilibrium prices. As each agent's demand depends only on the average price, $\overline{p}$ and their own price, for convenience, we refer to a set of prices as being in equilibrium when the price assigned to each agent is optimal for the agent given the resulting average price, $\overline{p}$. Similar methods have been used in the literature in the context of static non-atomic games (Aumann 1964, Schmeidler 1973, Ostroy and Zame 1994) and dynamic mean field games (Lasry and Lions 2007, Olszewski and Siegel 2016, Carmona and Wang 2021, Light and Weintraub 2022).

In any equilibrium, the set of active agents have a cost no greater than c_h, and the remaining agents with greater costs are inactive (i.e., serve no demand). We focus on markets for which the overall level of demand is sufficiently price sensitive, $\beta > 1$, so that some agents choose to be inactive (i.e., $c_h < 1$). Our results qualitatively generalize to all β.

4 Centralized Pricing Equilibrium

With centralized pricing, the platform maintains control of pricing but cannot distinguish between agents whose costs remain private. As a result, the platform selects a single price, p, that applies to all agents based on the overall distribution of costs in the market.

Agents only decide whether to participate in the market. The price equilibrium is straightforward: each agent correctly expects p to be the average price on the platform, $p = \overline{p}$. In turn, all agents have demand

$$q(\overline{p}) = 1 - \beta\overline{p}.$$

Accounting for the commission the agent pays the platform, ϕ, an agent with cost c earns $\pi_c(\overline{p})$,

$$\pi_c(\overline{p}) = q(\overline{p})((1-\phi)\overline{p} - c).$$

The highest cost agent that is active in the market, c_h, has no earnings, $\pi_{c_h}(\overline{p}) = 0$, which implies

$$c_h = (1-\phi)\overline{p}.$$

The platform selects the commission rate, ϕ, and the price, $\overline{p}$ to maximize its profit, $\Pi^{\mathcal{C}}$, given the anticipated participation decisions of the agents:

$$\begin{aligned} \max_{\overline{p},\phi} \quad & \Pi^{\mathcal{C}} = \phi\overline{p}\int_0^{c_h} (1-\beta\overline{p})\, dc \\ \text{s.t.} \quad & c_h = (1-\phi)\overline{p}. \end{aligned}$$

From Proposition 1, there is a unique optimal price and commission for the platform.

Proposition 1. *With centralized pricing, there exists a unique optimal price,* $\overline{p}^* = 2/(3\beta)$, *and commission rate,* $\phi^* = 1/2$, *for the platform. Table 1 summarizes the equilibrium metrics.*

Under centralized pricing, the platform completely eliminates competition among the agents by selecting a single price, so the degree of platform competition, γ, does not influence the equilibrium outcome. While the platform has full control over the average price, $\overline{p}$, a single price eliminates the benefit of tailoring prices to the agents' varying costs. As a result, some agents serve less demand than they would desire, while others prefer to leave the market entirely, though they would have stayed had they been given the freedom to set their own prices. Although agents only decide on participation, they retain positive earnings (one third of the total) because cost heterogeneity gives them some market power.

	Centralized Pricing With Commission Fees (§4)	Decentralized Pricing With Commission Fees (§5)	Optimal Design(§7.1)
Agent prices, $p^*(c)$	$\frac{2}{3\beta}$	$\frac{3}{2}\left(\frac{1}{2\beta+\gamma}\right)+c$	$\frac{4\gamma-\beta}{6\gamma\beta}+\frac{\beta}{\gamma}c$
Average market price, $\overline{p}^*$	$\frac{2}{3\beta}$	$\frac{2}{2\beta+\gamma}$	$\frac{2}{3\beta}$
Agent quantities, $q^*(c)$	$\frac{1}{3}$	$\frac{3}{2}\left(\frac{\gamma}{2\beta+\gamma}\right)-\gamma c$	$\frac{1}{2}-\beta c$
Total quantity served, Q^*	$\frac{1}{9\beta}$	$\frac{9}{8}\left(\frac{\gamma}{(2\beta+\gamma)^2}\right)$	$\frac{1}{8\beta}$
Supply of agents, c_h^*	$\frac{1}{3\beta}$	$\frac{3}{2}\left(\frac{1}{2\beta+\gamma}\right)$	$\frac{1}{2\beta}$
Platform's profit, Π^*	$\frac{1}{27\beta^2}$	$\frac{9}{8}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right)$	$\frac{1}{24\beta^2}$
Agents' total earnings, π^*	$\frac{1}{54\beta^2}$	$\frac{9}{16}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right)$	$\frac{1}{48\beta^2}$

Table 1: Equilibrium market outcomes with three platform designs: centralized pricing with a commission fee (§4), decentralized pricing with a commission fee (§5), and the platform's optimal design (§7.1).

5 Decentralized Pricing Equilibrium

When the platform opts for decentralized pricing, agents select their own price. This is a complex decision, as it depends on the prices the other agents choose. As discussed in §3, a price equilibrium occurs when all agents select optimal prices given the equilibrium prices of others.

As each agent's demand depends on the other agents' price choice only through the aggregate price metric, $\overline{p}$, a price equilibrium can be defined in terms of the average price on the platform. Let $\overline{p}_e$ be an agent's expectation for the average price in equilibrium. Let $p^*(c, \overline{p}_e)$ be the price selected by an agent with cost c, that maximizes the agent's earnings,

$$p^*(c, \overline{p}_e) = \arg\max_p q(p, \overline{p}_e)((1-\phi)p - c).$$

Let $R(\overline{p}_e, \overline{p})$ be the total revenue on the platform given the price expectation of the agents and the actual average price, $\overline{p}$:

$$R(\overline{p}_e, \overline{p}) = \int_0^{c_h} q(p^*(c, \overline{p}_e), \overline{p}) p^*(c, \overline{p}_e) dc.$$

Let $Q(\overline{p}_e, \overline{p})$ be the total quantity served:

$$Q(\overline{p}_e, \overline{p}) = \int_0^{c_h} q(p^*(c, \overline{p}_e), \overline{p}) dc.$$

The actual average price on the platform is the ratio of total revenue to total quantity. In equilibrium, the actual average price on the platform must match the agents' expected average price, $\overline{p}_e = \overline{p}$, meaning

$$\overline{p} = \frac{R(\overline{p}, \overline{p})}{Q(\overline{p}, \overline{p})} = \frac{\int_0^{c_h} q(p^*(c, \overline{p}), \overline{p}) p^*(c, \overline{p}) dc}{\int_0^{c_h} q(p^*(c, \overline{p}), \overline{p}) dc}. \tag{2}$$

The existence and uniqueness of a price equilibrium is not assured under any fee structure. However, Proposition 2 establishes that within the broad class of fee structures that are affine in price, there indeed exists a unique price equilibrium. The commission fee clearly resides within this class, and so do the platform optimal fees considered in Section 7.1.

Proposition 2. *For any fee that is affine in price (i.e., the fee paid per unit served includes two components: (i) a fixed base; and (ii) a component linear in the agent's price), there exists a unique price equilibrium with decentralized pricing.*

Next, we proceed to determine the platform's optimal commission fee and the resulting price equilibrium. The earnings of an agent with cost c that selects price p and has an average price expectation $\overline{p}$ is

$$\pi_c(p, \overline{p}) = \big(1 - \beta\overline{p} + \gamma(\overline{p} - p)\big)((1 - \phi)p - c).$$

An agent's earnings is concave in price, so we obtain the agent's unique optimal price,

$$p^*(c, \overline{p}) = \frac{1}{2}\left(\frac{1 - (\beta - \gamma)\overline{p}}{\gamma} + \frac{c}{1 - \phi}\right). \tag{3}$$

The price an agent chooses, (3), depends on two components. The first is the base amount of demand available to all agents. The second depends on the agent's own cost and the commission.

Because they have heterogeneous prices and costs, agents have heterogeneous earnings:

$$\pi_c(p^*(c, \overline{p}), \overline{p}) = \frac{((1 - \phi)(1 - (\beta - \gamma)\overline{p}) - c\gamma)^2}{4\gamma(1 - \phi)}.$$

The active agent with the highest cost, c_h, is

$$c_h = \frac{(1-\phi)(1-(\beta-\gamma)\overline{p})}{\gamma}. \tag{4}$$

The platform anticipates the agents' responses given in (3) and (4), and chooses the commission to maximize its profit, $\Pi^{\mathcal{D}}$:

$$\begin{aligned} \max_{\phi} \quad & \Pi^{\mathcal{D}} = \phi\overline{p}\int_0^{c_h} q(p^*(c,\overline{p}),\overline{p})\, dc \\ \text{s.t.} \quad & c_h = \frac{(1-\phi)(1-(\beta-\gamma)\overline{p})}{\gamma} \\ & \text{Eq. (2).} \end{aligned}$$

Proposition 3 provides the platform's unique optimal commission.

Proposition 3. *With decentralized pricing, the platform's unique optimal commission is* $\phi = 1/2$. *Table 1 summarizes the equilibrium metrics.*

With decentralized pricing, there is price dispersion on the platform: agents choose prices that are linear in their private costs. The platform can only influence these prices indirectly, through its selection of the commission. Consequently, the prices in the market are sensitive to the level of competition between agents, γ.

6 When to Delegate Pricing to Agents?

Figure 2 illustrates when a platform using a commission fee should select the single price (centralized pricing) and when the platform should allow agents to select their own prices (decentralized pricing). In short, it is best to let agents select their own prices when the competitive effect (γ) is neither too strong nor too weak. In those situations, competition among the agents produces reasonable prices, so the platform is better off letting agents tailor their prices to their costs. However, when agents are able to compete aggressively (large γ) or competition is weak (low γ), the platform is better off grabbing the pricing reins to avoid prices that spiral too low or too high. Proposition 4 formalizes this result.

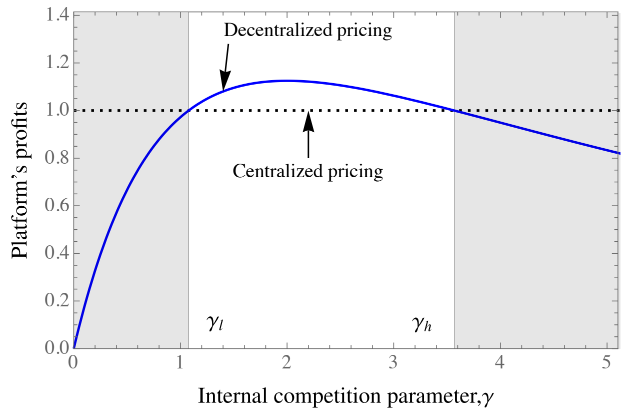

Figure 2: The platform's profit with decentralized pricing (solid line), as a fraction of the platform's profit with centralized pricing (scaled to 1) with $\beta = 2$. Shaded area: centralized pricing is preferred. White area: decentralized pricing is preferred.

Proposition 4. *There exists $\gamma_l < \beta$ and $\beta < \gamma_h$ such that the platform prefers decentralized pricing when $\gamma_l < \gamma < \gamma_h$, and otherwise prefers centralized pricing. Decentralized pricing is most preferred when $\beta = \gamma$.*

According to Proposition 4 the advantage of decentralized pricing is greatest when $\gamma = \beta$, in which case decentralized pricing yields 12.5% higher revenue for the platform than centralized pricing. In this special situation, there is no competition among the agents, i.e., they are effectively local monopolies. Because there is no competition among the agents, regulating competition among the agents is not important for the platform. Instead, the platform uses decentralized pricing to reap the benefits of allowing the agents to tailor their prices to their costs.

As γ deviates from β, in either direction, the platform's earnings with decentralized pricing decrease. When $\gamma < \beta$ the platform attractiveness effect dominates, and the average price in the market, $\overline{p}$, is higher than the price the platform would select when it controls pricing. Agents too would prefer they collectively lower their prices, but because all individual agents are powerless to influence the average price, and their ability to steal market share is weak, prices remain high. On the other hand, when $\beta < \gamma$, the agent competition effect dominates platform attractiveness. Now, each agent aggressively lowers its price to try to capture market share. With all agents trying to take share, the low average price attracts demand to the platform but also causes some agents to become inactive. In this case the platform would prefer higher prices.

In sum, the commission fee is insufficient to adequately regulate competition. When the agent competition effect is either too weak $\gamma < \gamma_l$ or too strong $\gamma_h < \gamma$, it is best for the platform to use centralized pricing to shut off all competition among the agents.

Agent Earnings

From Table 1, total agent earnings remain a fixed fraction (one half) of the platform's profit in either pricing regime (centralized or decentralized) and no matter the demand characteristics (β and γ). Consequently, the platform's goal to maximize its own profit is actually aligned with the objective to maximize total agent earnings. However, agents with a particular cost may have a preference over the pricing policy (Corollary 1).

Corollary 1. *When the platform finds it optimal to switch from centralized to decentralized pricing (that is, $\gamma_l < \gamma < \gamma_h$): (i) total agent earnings increase, but (ii) some agents can be worse off.*

Even when decentralized pricing increases total agent earnings, Figure 3 illustrates that agents with certain costs can be worse off. For example, with weak agent competition (left figure), many low-cost agents earn more with centralized pricing (dashed curve) than with decentralization pricing (blue curve). In contrast, some high-cost agents who were crowded out with centralized pricing are able to profitably participate in the market when they can select their own price. With stronger competition (right figure), agents with intermediate costs prefer centralized pricing, while the rest are better off with decentralized pricing. Although there is no clear preference over the pricing regime for agents with a certain cost, regulatory agencies typically focus on total earnings. Furthermore, if an agent's cost is primarily an opportunity cost, it is possible that an agent's cost could vary over multiple participation sessions on the platform. In such case, each agent may be more focused on the average earnings across all cost levels rather than the earnings of a specific cost.

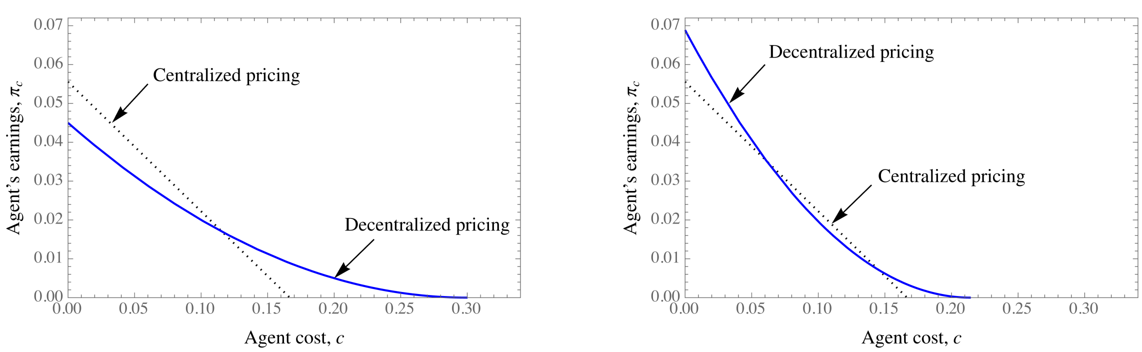

Figure 3: Profit of agent with cost c under centralized pricing (dotted line) and decentralized pricing with commission (solid line) for $\beta = 2$, $\gamma = 1$ (left) and $\beta = 2$, $\gamma = 3$ (right).

7 Optimal Platform Design

Given that neither price control policy is preferred in all settings (Section 6), there could be better designs for the platform. In this section we identify three such alternatives, all of which maximize the platform's profit. They differ in how they would be implemented in practice. With the first, the agents report their costs and from the reported costs the platform assigns prices and fees for the agents to pay. With the second and third the agents choose their prices, and the fees are affine functions of easily observable metrics: the agent's price or the agent's demand quantity. As the latter two mechanisms implement decentralized pricing, they satisfy regulatory requirements for agents to maintain control over their prices.

7.1 Optimal Centralized Pricing

According to the revelation principle (Myerson 1981), the platform's optimal design (i.e., pricing policy and fee structure) resides within the set of truth-inducing mechanisms. With those mechanisms, each agent reports a cost, $\tilde{c}$, and the platform announces a menu that maps the set of possibly reported costs into a price posted for the agent, $p(\tilde{c})$, and a per-unit served fee, $f(\tilde{c})$, the agent pays the platform. Consequently, we refer to this as a "cost-based" fee structure. This mechanism is truth inducing if (i) it is optimal for each agent to report their cost truthfully, i.e., $\tilde{c} = c$ for all c (assuming all other agents do so as well) and (ii) an agent's earnings from participation in the platform is at least equal to the agent's best outside option (which is normalized to zero). The first requirement is referred to as the incentive compatibility constraint and the second is referred to as the individual rationality constraint.

Let $\pi_c(\tilde{c})$ be an agent's earning with cost c that reports cost $\tilde{c}$ to the platform:

$$\pi_c(\tilde{c}) = q(p(\tilde{c}), \overline{p})(p(\tilde{c}) - c - f(\tilde{c})).$$

The platform's optimal mechanism can be found through the following optimization problem:

$$\begin{aligned} \max_{p(c), f(c)} \quad & \Pi = \int_0^{c_h} q(p(c), \overline{p}) f(c) \, dc \\ \text{s.t.} \quad & \pi_c(c) \geq \pi_c(\tilde{c}) \ , \forall c \in [0,1], \forall \tilde{c} \in [0,1] \\ & \pi_c(c) \geq 0 \ , \forall c \in [0,1] \\ & \text{Eq. (2)} \end{aligned}$$

where, recall, c_h is the highest cost among the agents active on the platform.

There are several challenges to derive the optimal mechanism. First, there is no constraint on the form of the fee structure, $f(c)$. Second, an agent's earnings depend on all of the prices in the market through the average price, $\overline{p}$. Hence, the platform's mechanism must have consistent expectations such that given the equilibrium cost reports, the assigned prices are such that the average price leads to the expected quantities and profit.

To overcome these challenges, a transformation of the payoffs is used to decouple the initial design problem into a set of independent designs across all agents. From this emerges the form of the payment structure and the optimal mechanism can be evaluated in closed form, Proposition 5. To the best of our knowledge, this is the first application of optimal mechanism design to a platform with differentiated competition.

Proposition 5. *The platform maximizes its profit by asking all agents to report their costs and then an agent with cost $\tilde{c}$ is assigned price $p(\tilde{c})$ and pays the per-unit fee $f(\tilde{c})$ to the platform, where $p(\tilde{c}) = \frac{2}{3\beta} - \frac{1}{6\gamma} + \frac{\beta}{\gamma}\tilde{c}$, $f(\tilde{c}) = \frac{1}{12}\left(\frac{5}{\beta} - \frac{2}{\gamma}\right) + \left(\frac{\beta}{\gamma} - \frac{1}{2}\right)\tilde{c}$. With this mechanism each agent in equilibrium reports its cost correctly, $\tilde{c} = c$, i.e., this mechanism is truth-inducing. See Table 1 for outcome metrics of the optimal mechanism.*

A comparison of the platform's optimal mechanism with commission fees and centralized or decentralized pricing reveals the limitations of those strategies. From Table 1, centralized pricing with commission fees achieves the optimal average price but it does not create the necessary dispersion in prices. Consequently, with centralized pricing there are fewer than the optimal number of agents (high cost agents are excluded from the market) and the total supply is less than ideal (low cost agents are not supplying as much as they would want). In other words, because centralized pricing lacks the ability to tailor prices to costs, it leads to sub-optimal supply.

Decentralized pricing with a commission fee addresses the main limitation of centralized pricing, i.e., it allows prices to be tailored to the agents' costs. But with a simple commission fee, the tailoring is insufficient. To explain, with just a commission fee, price increases linearly in cost as follows,

$$\frac{\partial p(c)}{\partial c} = 1.$$

With the optimal fee structure, the responsiveness of prices to costs depends on market characteristics:

$$\frac{\partial p(c)}{\partial c} = \frac{\beta}{\gamma}.$$

In particular, when platform competition is relatively weak, $\gamma < \beta$, then optimal prices are more sensitive to agent costs with the optimal price schedule. In this situation, decentralized pricing with a commission fee leads to an average price which is too high, so the optimal design skews prices lower for the low cost agents. In contrast, when platform competition is relatively strong,

$\beta < \gamma$, the optimal design needs to dampen price competition. It does so through prices that are less responsive to the agents' costs.

In sum, the platform's optimal design addresses the limitation of centralized pricing (prices are not tailored to agents' costs) and the limitation of decentralized pricing (agents competing on their own generally do not yield the correct average price for the market, nor the correct set of active agents). Interestingly, even the agents, as a group, are better off with the platform's optimal design. Hence, the best design for the platform increases the platform's profit by eliminating the inefficiencies of poorly controlled agent competition rather than by stealing a larger share of the available wealth.

7.2 Optimal Decentralized Pricing

The cost-based fee structure is impractical in the sense that it relies on agents to report costs and its centralized structure is likely to fail to meet regulatory price control requirements. A preferable alternative grants agents autonomy, uses observable metrics to decide payments, and continues to optimize the platform's profit. This section introduces two mechanisms that achieve those goals. They work because both price and quantity are proxies for the agents' unobservable costs. Their simplicity enables implementation in the field, evidenced in actual practice.

7.2.1 Decentralized Commission-Plus Fees

Consider a mechanism in which the platform allows decentralized pricing and charges agents per unit of demand served a price-based commission, ϕ, and a fixed base fee, w. The platform's total fee per unit of demand served is $f(p)$,

$$f(p) = \phi p + w.$$

We refer to this decentralized pricing fee structure as *commission-plus* - it is a commission fee with a base minimum fee per unit served, w.

An agent with cost c that posts price p and has an average price expectation $\overline{p}$ earns

$$\pi_c(p, \overline{p}) = q(p, \overline{p})((1 - \phi)p - c - w).$$

The optimal price for this agent is

$$p^*(c, \overline{p}) = \frac{1}{2}\left(\frac{1 - (\beta - \gamma)\overline{p}}{\gamma} + \frac{c + w}{1 - \phi}\right).$$

For any average price, $\overline{p}$, only agents with costs less than the threshold c_h are active on the platform, where $\pi_{c_h}(p^*(c_h, \overline{p}), \overline{p}) = 0$, and

$$c_h = c_h(\phi, w, \overline{p}) = \frac{(1-\phi)(1-(\beta-\gamma)\overline{p})}{\gamma} - w.$$

The platform's profit is

$$\Pi^{\mathcal{C}+}(\phi, w) = (\phi\overline{p} + w) \int_0^{c_h(\phi,w,\overline{p})} q(p^*(c, \overline{p}), \overline{p})\, dc.$$

In a price equilibrium, the realized average price is consistent with the prices the agents select and matches their expectations for the average price:

$$\overline{p} = \frac{\int_0^{c_h(\phi,w,\overline{p})} q(p^*(c,\overline{p}),\overline{p})p^*(c,\overline{p})\, dc}{\int_0^{c_h(\phi,w,\overline{p})} q(p^*(c,\overline{p}),\overline{p})\, dc}.$$

From Proposition 6, the platform can use a unique commission-plus fee structure to replicate the profits of the optimal centralized cost-based fee, and this is done with decentralized pricing to satisfy regulatory concerns.

Proposition 6. *With decentralized pricing, there exists a unique commission-plus fee structure that yields the platform's optimal profit in equilibrium. Furthermore, with these fees, the prices, quantities served and earnings are the same for all agents and the platform as with the centralized pricing optimal design from Proposition 5. The payment terms of this fee structure are given in Table 2.*

Affine contract $f^*(x) = \phi^* \cdot x + w^*$	Cost Reporting §7.1 (Centralized)	Commission-plus §7.2.1 (Decentralized)	Quantity-based §7.2.2 (Decentralized)
Variable fee, ϕ^*	$\frac{\beta}{\gamma} - \frac{1}{2}$	$1 - \frac{\gamma}{2\beta}$	$\frac{1}{2\beta} - \frac{1}{\gamma}$
Fixed fee, w^*	$\frac{1}{12}\left(\frac{5}{\beta} - \frac{2}{\gamma}\right)$	$\frac{\gamma}{3\beta^2} - \frac{1}{3\beta}$	$\frac{1}{6\beta} + \frac{1}{3\gamma}$

Table 2: Equilibrium variable (ϕ^*) and fixed (w^*) fees for the three optimal affine fee structures $f^*(x) = \phi^* \cdot x + w^*$: cost reporting in §7.1 ($x = c$), commission-plus in §7.2.1 ($x = p$), and quantity-based in §7.2.2 ($x = q$).

The commission-plus fee structure achieves the platform's optimal profit because each component of the fee is tuned to address different control issues. The commission portion of the fee, ϕ, regulates the responsiveness of prices to the agents' costs:

$$\frac{\partial p^*(c,\overline{p})}{\partial c} = \frac{1}{2}\frac{1}{1-\phi} = \frac{\beta}{\gamma}c.$$

The fixed per-unit fee the agents pay, w, allows the platform to adjust the average price in the market independently from the dispersion (i.e., slope) of prices, i.e., a change in the fixed component, w, causes all agents, independent of their cost, to shift their prices by the same amount:

$$\frac{\partial p^*(c,\overline{p})}{\partial w} = \frac{1}{2(1-\phi)}.$$

Recall, the base "fee", w, need not be positive, i.e., an actual fee. It is indeed an actual fee, $0 < w$, when competition is strong on the platform, $\beta < \gamma$. In these situations, decentralized pricing with a commission leads to prices that are too low and too few active agents, reducing the overall revenue available on the platform. Hence, the platform charges a fixed fee per unit served, $0 < w$, to force all agents to raise their prices. Doing so returns the average price on the platform to the desirable level. On the other hand, when competition is weak on the platform, $\gamma < \beta$, the commission fee leads to prices that are too high. The commission-plus fee addresses this issue by converting w from a fee into a subsidy, $w < 0$, which motivates the agents to serve more demand, i.e., to lower their prices. Put differently, the subsidy in essence lowers each agent's cost, which causes the agents to lower their prices, all by the same amount.

7.2.2 Decentralized Quantity-Based Fees

Quantity-based fee structures have been shown to help the efficiency of decentralized supply chains (e.g., Monahan 1984, Weng 1995, Corbett and De Groote 2000). However, to the best of our knowledge, their use for price coordination has not been studied in the context of service platforms.

Consider a quantity-based fee structure in which an agent that serves q units of demand pays the platform $f(q)$ per unit, where $f(q)$ includes a fixed amount per unit, w, and a variable amount that depends on the quantity served, ϕq,

$$f(q) = \phi q + w.$$

An agent with cost c that selects price p and has an average price expectation $\overline{p}$ earns:

$$\pi_c(p,\overline{p}) = q(p,\overline{p})(p - c - \phi q(p,\overline{p}) - w).$$

The optimal price of the agent is

$$p^*(c,\overline{p}) = \frac{1}{2(1+\gamma\phi)}\left(\frac{1-(\beta-\gamma)\overline{p}}{\gamma} + c + 2\phi + w\right).$$

Only agents with sufficiently low costs participate. Let c_h be the highest cost agent that chooses to participate on the platform, i.e., $\pi_{c_h}(p^*(c_h,\overline{p}),\overline{p}) = 0$, which implies

$$c_h = c_h(\phi, w, \overline{p}) = \frac{1-(\beta-\gamma)\overline{p}}{\gamma} - w.$$

The platform's profit is

$$\Pi^Q(\phi, w) = \int_0^{c_h(\phi,w,\overline{p})} q(p^*(c,\overline{p}),\overline{p})(\phi q(p^*(c,\overline{p}),\overline{p}) + w)\, dc.$$

Proposition 7 shows that decentralized pricing with a quantity-based fee structure can also maximize the platform's profit.

Proposition 7. *With decentralized pricing, there exists a unique quantity-based fee structure that yields the platform's optimal profit in equilibrium. Furthermore, with these fees, the prices, quantities served and earnings are the same for all agents and the platform as with the platform's optimal commission-plus fee structure with decentralized pricing discussed in Proposition 6. The payment terms of this fee structure is given in Table 2.*

As with commission-plus fees, the two components of the quantity-based fees have different functions. The quantity parameter, ϕ, is responsible for regulating price dispersion (or, more precisely, the sensitivity of selected prices to agents' costs):

$$\frac{\partial p^*(c,\overline{p})}{\partial c} = \frac{1}{2(1+\phi\gamma)} = \frac{\beta}{\gamma}.$$

In most circumstances, a quantity discount is offered: when $\gamma < 2\beta$, the per-unit fee an agent pays is decreasing in the quantity the agent serves, $\phi < 0$. But if competition is particularly intense on the platform, $2\beta < \gamma$, then it is actually necessary for the platform to switch to surcharging agents for greater quantities to motivate them to raise their prices. The base fee component, w, is responsible for regulating the average price. As the variable component, ϕ, also impacts the price level, unlike with commission-plus fees, the fixed base fee, w, never converts to a subsidy.

Like the commission-plus fee, the quantity-based fee is able to replicate the optimal mechanism because agents with different costs naturally want to select different prices, which means they serve different quantities.

8 Extensions and Robustness

The primary insights from the main model continue to hold with several extensions.

8.1 Capacity Constraints

Agents may have a constraint on the amount of demand that can be served, but the commission-plus fee structure remains optimal for the platform (Proposition 8). Naturally, capacity constraints are never desirable for the platform when the platform can use an optimal design. However, with the simpler commission fee structure, capacity constraints can be advantageous to the platform. Rather than use its fee structure to dampen competition, the platform can lean on the capacity constraints to do so.

Proposition 8. *Let k be the maximum demand an agent can serve. Given this capacity constraint, the platform achieves its optimal profit with decentralized pricing and a commission-plus fee structure,* $\phi^* = 1 - \frac{\gamma}{2\beta}$, $w^* = \left(\frac{4k^2-6k+3}{1-k}\right)\left(\frac{\gamma-\beta}{6\beta^2}\right)$.

8.2 Alternative Cost Distribution

The uniform distribution facilitates analytical results and yields an optimal design that can be implemented with a relatively simple structure. For other agent cost distributions, prices are non-linear in the agents' costs in the optimal design. Hence, the optimal fee structure cannot be implemented with just a commission and per-unit fees (see Online Appendix, page 53). Nevertheless, as reported elsewhere (e.g., Cachon and Zhang 2006) the optimal version of decentralized pricing with commission-plus may perform well.

Consider a Beta(α_1, α_2) distribution with $\alpha_1 = \alpha_2 = \alpha$, which implies the mean is a constant 0.5 and the density function is symmetric about the mean. Although closed form solutions for the optimal fees are not available, optimal fees are numerically evaluated for 400 scenarios: $\gamma \in \{0.1, 0.2, ..., 10\}$, $\beta \in \{2\}$, and $\alpha \in \{2, 3, ..., 5\}$. The optimal linear fee yields on average 99.98% of the optimal profit for the platform and no less than 99.9%. These numeric results suggest the commission-plus fee structure can be robust to variations in the agents' cost distribution, even distributions that are far from uniform.

8.3 Throughput Maximization

Platforms may prioritize maximizing throughput over profits, which has been considered in other models (e.g., Ahmadinejad et al. 2019, Castro et al. 2020, Yan et al. 2020). As profit is the product of quantity and margin, maximizing throughput (i.e., quantity) is a related objective. Hence, a platform can also use decentralized pricing and a commission-plus fee structure to maximize the total quantity served in the market (see Online Appendix, page 58).

9 Conclusion

Who should control pricing on a service platform and how should fees be collected? When pricing is centralized with the platform, the platform is able to perfectly regulate the average price in the market and this significantly influences the overall demand the platform is able to attract. However, because agents know their own costs to participate, they are best able to tailor their price to their circumstances. The issue with decentralized pricing is that the competition among the agents may not lead to the appropriate prices nor the best set of active agents in the market. Depending on the intensity of competition, prices can be too high or too low, and either some agents are active in the market when they should not be, or some choose to not participate when they should. Hence, the proper regulation of competition on the platform is critical.

Commission fees are simple to explain and administer, but lack sufficient precision on their own to properly control prices. Consequently, if the platform is restricted to just commission fees, then it may prefer to opt for centralized pricing to avoid the adverse scenarios of decentralized pricing.

Fortunately, it is possible to combine the robustness of centralized pricing with the advantages of decentralization using as simple fee structure: the platform should charge a per-unit served fee that consists of a fixed component and another component either based on the agent's price (commission-plus) or the amount of demand the agent serves (quantity fee). These refined fee structures allow the platform to influence prices in a way that is not too heavy-handed, maintaining the flexibility for agents to set prices that reflect their costs. In fact, they yield the platform's optimal profit. Furthermore, even the agents earn more with these affine fee structures than with either centralized pricing or decentralized pricing and a basic commission.

So the question for a service platform is not so much who should control pricing, but how they do it. For example, decentralized pricing may be necessary to classify agents as contractors rather than employees. Although that classification has implications for who is willing to work on a platform and the costs of their work, it may be possible to give agents full pricing control and yet fully correct for the potentially negative consequences of doing so.

Limitations

The primary trade-off we bring to light, between the costs of information asymmetry in centralized pricing and the coordination challenges in decentralized settings, appears robust against the backdrop of stylized assumptions. However, as is the case in models of markets generally, the exact formulation of the most effective fees is inherently interwoven with the choice of the consumer demand model. Our analysis is predicated on a conventional demand model that is linear in prices, which naturally complements the affine fee structures we propose. Such linear models are well established in the literature (Angeletos and Pavan 2007, Bimpikis et al. 2019, Feldman et al. 2023). Nonetheless, exploring demand models that introduce non-linear dynamics could be a fruitful direction for future research.

In the context of our study, the robustness tests outlined in Section 8 substantiate the practicality of affine fee structures. Even when these structures fall short of absolute optimality, they often deliver outcomes that are very close to the ideal. Expanding future research could investigate the robustness of affine fees to settings that include risk aversion, moral hazard, explicit forms of horizontal differentiation, and bounded rationality. Lastly, while we concentrate on pricing, platforms typically face a multitude of strategic decisions. The potential for optimal decentralized fee structures to enhance overall outcomes in a multi-decision context is another promising topic for investigation.

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Appendix

A Demand Model

This section derives our demand model from a customer choice process and relates it to an established model in the literature.

The "single representative consumer" (SC) approach is well established in the literature to derive a demand model from a choice process across differentiated agents in oligopoly settings (Singh and Vives (1984) and Ledvina and Sircar (2012)). With this approach, given the agents' prices, a single consumer selects quantities across all agents to maximize a global linear-quadratic utility, U,

$$U = \frac{1}{\beta}\int_0^1 q(c)dc - \frac{1}{2\gamma}\int_0^1 q(c)^2 dc - \frac{1}{2}\left(\frac{1}{\beta} - \frac{1}{\gamma}\right)\int_0^1 q(c)\left(\int_0^1 q(x)dx\right)dc - \int_0^1 p(c)q(c)dc,$$

where $q(c)$ is the demand quantity selected for an agent with cost $c \in [0,1]$. The model is usually presented in discrete form, but here we present the analogous continuous form used in Foster et al. (2008). Define an active agent to be one with positive quantity, $q(c) > 0$. To maximize U, the quantity of an active agent satisfies $\partial U/\partial q(c) = 0$, or

$$0 = \frac{1}{\beta} - \frac{1}{\gamma}q(c) - \left(\frac{1}{\beta} - \frac{1}{\gamma}\right)\int_0^1 q(c)dc - p(c). \tag{5}$$

The quantity demanded from an agent with cost c, $q(c)$, depends on the agent's price, $p(c)$, and an aggregate metric of quantity, which in this case is the total quantity in the market. If all agents are active, integrating the marginal utilities in (5) over all active agents, we obtain

$$\begin{aligned} 0 &= \int_0^1 \left(\frac{1}{\beta} - \frac{1}{\gamma}q(c) - \left(\frac{1}{\beta} - \frac{1}{\gamma}\right)\int_0^1 q(c)dc - p(c)\right) dc \\ &= \frac{1}{\beta} - \frac{1}{\gamma}\int_0^1 q(c)\, dc - \left(\frac{1}{\beta} - \frac{1}{\gamma}\right)\int_0^1 q(c)dc - \int_0^1 p(c)\, dc, \end{aligned}$$

which simplifies to

$$\int_0^1 q(c)\, dc = 1 - \beta\int_0^1 p(c)\, dc.$$

By substituting this relationship into (5) and solving for $q(c)$, we express each active agent's demand in terms of the average price of all agents and the agent's own price:

$$q(c) = 1 - (\beta - \gamma)\int_0^1 p(c)dc - \gamma p(c). \tag{6}$$

Instead of a single representative consumer, consider a demand model derived from a continuum of consumers (CC). In particular, there is one consumer for each agent c that selects the agent's quantity, q, to maximize a utility, $\hat{u}_c(q)$, derived from the agent's good and one other good which is a composite of all other goods. In effect, each agent offers a good on the market which competes with all other agents through the composite good. In this case,

$$\hat{u}_c(q) = \frac{1}{\beta}q - \frac{1}{2\gamma}q^2 - \left(\frac{1}{\beta} - \frac{1}{\gamma}\right) q\hat{q} - p(c)q,$$

where

$$\hat{q} = \int_0^1 q(c)dc.$$

The utility $\hat{u}_c(q)$ a consumer receives from the agent's offering has several components. The first two terms reflect the utility from the agent's good in isolation (with the usual declining marginal returns to consumption). The third term indicates that the agent's good is more or less valuable depending on the magnitude of the composite good, which is proportional to the average quantity in the market, $\hat{q}$. (As there is a unit mass, the average and the total quantity in the market are identical.) The fourth term is the payment made to the agent.

As all agents are small, the consumer of each agent is taken to be small, which means their choice of q has no impact on the composite good, i.e., each agent takes $\hat{q}$ as given. It follows that the quantity selected for an agent with cost c satisfies $\partial\hat{u}_c(q(c))/\partial q(c) = 0$, or

$$0 = \frac{1}{\beta} - \frac{1}{\gamma}q(c) - \left(\frac{1}{\beta} - \frac{1}{\gamma}\right) \hat{q} - p(c),$$

which is equivalent to (5). Thus, the CC model yields the same demand functions as the SC model, (6). In other words, the demand model that is typically used in the literature, which has been derived from the SC utility, can also be obtained from the utility maximization of a continuum of consumers.

A limitation of these models, as noted by Ledvina and Sircar (2012), is that each agent's demand can be expressed only in terms of prices conditional on the set of active agents. This is not problematic if all agents are assumed to be active for all sets of feasible prices. But it is a concern with markets in which some agents choose to not participate in some circumstances (e.g., if other agents choose very low prices). To address this issue, take the continuum of consumers approach for deriving the demand function and make only one modification. Instead of using the total quantity in the market as the aggregate metric for the composite good, $\hat{q}$, use the weighted average quantity,

$\overline{q}$,

$$\overline{q} = \int_0^1 \left(\frac{q(c)}{\int_0^1 q(c)dc} \right) q(c)dc.$$

In particular, let the utility earned from each agent with quantity q be

$$\overline{u}_c(q) = \frac{1}{\beta}q - \frac{1}{2\gamma}q^2 - \left(\frac{1}{\beta} - \frac{1}{\gamma}\right) q\overline{q} - p(c)q.$$

As in the CC model, the utility received from a agent's good depends on q, the agent's price and a composite good from the offerings of all other agents. The only difference is the construction of the composite good, either being the straight or weighted average of the agents' quantities.

We next confirm that the utility $\overline{u}_c$ of the continuum of consumers yields our demand model.

The consumer for each active agent selects a quantity to maximize their utility given $\overline{q}$. Hence, from the marginal utility for each agent, $\partial \overline{u}_c(q(c))/\partial q(c) = 0$, the optimal quantity satisfies

$$0 = \frac{1}{\beta} - \frac{1}{\gamma}q(c) - \left(\frac{1}{\beta} - \frac{1}{\gamma}\right) \overline{q} - p(c). \tag{7}$$

Inactive agents have zero quantities, $q(c) = 0$. Therefore,

$$0 = \left(\frac{1}{\beta} - \frac{1}{\gamma}q(c) - \left(\frac{1}{\beta} - \frac{1}{\gamma}\right) \overline{q} - p(c) \right) q(c) \tag{8}$$

is satisfied for all agents, including the inactive ones. Taking the integral of (8) over all agents, we obtain

$$\begin{aligned}
0 &= \int_0^1 \left(\frac{1}{\beta} - \frac{1}{\gamma}q(c) - \left(\frac{1}{\beta} - \frac{1}{\gamma}\right) \overline{q} - p(c) \right) q(c)\ dc \\
&= \frac{1}{\beta}\int_0^1 q(c)\ dc - \frac{1}{\gamma}\int_0^1 q(c)^2\ dc - \left(\frac{1}{\beta} - \frac{1}{\gamma}\right) \overline{q} \int_0^1 q(c)\ dc - \int_0^1 p(c)q(c)\ dc \\
&= \frac{1}{\beta}\int_0^1 q(c)\ dc - \frac{1}{\gamma}\int_0^1 q(c)^2\ dc - \left(\frac{1}{\beta} - \frac{1}{\gamma}\right) \left(\frac{\int_0^1 q(c)^2\ dc}{\int_0^1 q(c)\ dc} \right) \left(\int_0^1 q(c)\ dc \right) - \int_0^1 p(c)q(c)\ dc \\
&= \frac{1}{\beta}\int_0^1 q(c)\ dc - \frac{1}{\gamma}\int_0^1 q(c)^2\ dc - \left(\frac{1}{\beta} - \frac{1}{\gamma}\right) \int_0^1 q(c)^2\ dc - \int_0^1 p(c)q(c)\ dc,
\end{aligned}$$

which yields

$$\frac{\int_0^1 q(c)^2\ dc}{\int_0^1 q(c)\ dc} = 1 - \beta \left(\frac{\int_0^1 p(c)q(c)\ dc}{\int_0^1 q(c)\ dc} \right).$$

Rewriting the expression in terms of $\overline{q}$, we find

$$\overline{q} = 1 - \beta\overline{p}, \tag{9}$$

where $\overline{p}$ is the weighted average price,

$$\overline{p} = \int_0^1 \frac{q(c)}{\int_0^1 q(c)dc} p(c)dc.$$

Substitution of $\overline{q}$ from (9) into (7) yields our demand model for each agent:

$$q(c) = 1 - (\beta - \gamma)\overline{p} - \gamma p(c).$$

This demand for each agent c applies for all possible prices because the weighted average price naturally accounts for the agents that are not active (i.e., have zero quantity). Consequently, it is possible to compare two settings with different prices because both settings have the same coefficients in the demand model (i.e., β and γ).

Several features of the CC model with weighted average quantities are worth highlighting. First, as with the SC model, each agent's demand behaves as you would expect with respect to their own price and the aggregate metric. Second, the weighted average price ignores posted prices that do not lead to actual transactions and thus represents the average price actually paid on the platform. This is intuitively appealing: it is the price a consumer can expect to pay on the platform before the consumer observes the price they actually pay. Third, there is a precedent for the weighted average quantity. As in the well-known Herfindahl–Hirschman index, each agent's quantity is scaled by its market share,* the ratio of the agent's quantity to the total quantity. (The HH index further divides each agent's demand by the total quantity to normalize the index to range from 0 to 1, but that is not desirable here.) This ensures that the demand for each agent depends not only on the total quantity in the market but also the dispersion of that quantity, which is intuitively appealing. For example, a consumer's utility for chocolate may depend not only on the total quantity of other flavors but also on how that quantity is distributed among the other flavors. Fourth, and most importantly, because the coefficients of the demand model (β and γ) do not depend on the set of active agents (which is endogenous), "apples to apples" comparisons can be made across different fee structures and pricing regimes (e.g., Table 1).

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*We would like to acknowledge Robert Bray's insight to link our demand model to the Herfindahl–Hirschman index.

Pricing Control and Regulation on Online Service Platforms
Online Appendix

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B Proofs

Proof of Proposition 1. In centralized pricing, there exists a single price in the market. The average market price is equivalent to the price set by the platform. Let $\overline{p}$ be the price set and ϕ be the portion of revenue retained. Conditional on participation, an agent with cost c earns

$$\begin{aligned}\pi_c(\overline{p}) &= q(\overline{p})((1-\phi)\overline{p} - c) \\ &= (1-\beta\overline{p})((1-\phi)\overline{p} - c).\end{aligned}$$

Agent earnings are decreasing in cost and therefore there exists a threshold cost, c_h, for which agent with cost c participates if and only if $c \leq c_h$. The highest cost that participates is

$$\pi_{c_h}(\overline{p}) = 0 \implies c_h = c_h(\overline{p},\phi) = (1-\phi)\overline{p}.$$

Platform's profit maximization problem is

$$\begin{aligned}\max_{\overline{p},\phi} \quad \Pi^{\mathcal{C}}(\overline{p},\phi) &= \phi\overline{p} \int_0^{c_h(\overline{p},\phi)} q(\overline{p})\, dc \\ &= \phi(1-\phi)\overline{p}^2(1-\beta\overline{p}).\end{aligned}$$

The objective function is concave in ϕ and maximized at $\phi = 1/2$. Similarly, we have

$$\frac{\partial \Pi^{\mathcal{C}}(\overline{p},\phi)}{\partial \overline{p}} = \phi(1-\phi)\overline{p}(2-3\beta\overline{p}) \geq 0 \iff \overline{p} \in \left[0, \frac{2}{3\beta}\right].$$

Therefore, the platform's objective is maximized at

$$\begin{aligned}\overline{p} &= \frac{2}{3\beta}, \\ \phi &= \frac{1}{2}.\end{aligned}$$

Plugging in the equilibrium price and commission, the platform's profit is

$$\Pi^{\mathcal{C}} = \phi(1-\phi)\overline{p}^2(1-\beta\overline{p}) = \frac{1}{27\beta^2}.$$

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The highest cost that participates is

$$c_h = (1-\phi)\overline{p} = \frac{1}{3\beta}.$$

The total quantity of customers served in the market is

$$\begin{aligned} Q &= \int_0^{c_h(\overline{p},\phi)} q(\overline{p})\, dc \\ &= (1-\phi)\overline{p}(1-\beta\overline{p}) \\ &= \frac{1}{9\beta}. \end{aligned}$$

Agents' total earnings is

$$\begin{aligned} \pi^{\mathcal{C}} &= \int_0^{c_h(\overline{p},\phi)} q(\overline{p})((1-\phi)\overline{p}-c)\, dc \\ &= \frac{1}{2}(1-\beta\overline{p})\overline{p}^2(1-\phi)^2 \\ &= \frac{1}{54\beta^2}. \end{aligned}$$

□

Proof of Proposition 2. Consider a broad class of fee structures such that an agent pays the platform (i) a base unit fee, w, per unit served and (ii) a commission, ϕp, per unit served where $\phi < 1$ is the fixed commission rate and p is the agent's price. An agent with demand q pays the platform in total $q\,(w+\phi p)$. The fixed per-unit fee, w, can be negative, meaning that it is actually a per-unit subsidy.

With this payment structure, we seek to determine the uniqueness of a price equilibrium. Each agent's earnings depend on the average price on the platform, so it is sufficient to consider a price expectations equilibrium in which each agent expects the average price to be $\overline{p}_e$ and all agents select prices based on that expectation that yield $\overline{p}_e$ as the actual average price.

An agent with price expectation $\overline{p}_e$, cost c and price p expects to earn

$$\begin{aligned} \pi_c(p,\overline{p}_e) &= q(p,\overline{p}_e)\left((1-\phi)p-c-w\right) \\ &= \left(1-\beta\overline{p}_e+\gamma\left(\overline{p}_e-p\right)\right)\left((1-\phi)\,p-c-w\right). \end{aligned}$$

Because earnings are strictly concave in price, there exists a unique price, $p^*(c,\overline{p}_e)$, that maximizes the agent's earnings,

$$p^*(c,\overline{p}_e) = \frac{1}{2}\left(\frac{1-(\beta-\gamma)\,\overline{p}_e}{\gamma}+\frac{c+w}{1-\phi}\right). \tag{10}$$

Let the realized average price be $\overline{p}$. This agent then receives $q(p^*(c, \overline{p}_e), \overline{p})$ demand,

$$q(p^*(c, \overline{p}_e), \overline{p}) = \frac{1}{2}\left(1 + (\beta - \gamma)(\overline{p}_e - 2\overline{p}) - \frac{\gamma(c+w)}{1-\phi}\right)^+. \tag{11}$$

The realized quantity served is decreasing in cost, c. Define c_0 to be the smallest cost such that the optimal quantity is zero,

$$c_0 = \min\{c : q(p^*(c, \overline{p}_e), \overline{p}) = 0\} = \frac{(1-\phi)(1 + (\beta - \gamma)(\overline{p}_e - 2\overline{p})}{\gamma} - w. \tag{12}$$

The realized average price is

$$\overline{p} = \frac{\int_0^{\min\{c_0,1\}} q(p^*(c, \overline{p}_e), \overline{p}) p^*(c, \overline{p}_e)\, dc}{\int_0^{\min\{c_0,1\}} q(p^*(c, \overline{p}_e), \overline{p})\, dc}. \tag{13}$$

A price equilibrium exists if $\overline{p}_e = \overline{p}$. We will show that there can exist only one such equilibrium in any market.

From (12), given an equilibrium, i.e., $\overline{p} = \overline{p}_e$, the average price in equilibrium can be expressed in terms of c_0,

$$\overline{p} = \frac{\gamma(c_0 + w) + \phi - 1}{(1-\phi)(\gamma - \beta)}. \tag{14}$$

From (10), (11), (14) and $\overline{p}_e = \overline{p}$, the equilibrium condition (13) can be written as

$$\begin{aligned} 0 &= \frac{\gamma(c_0 + w) + \phi - 1}{(1-\phi)(\gamma - \beta)} - \frac{\int_0^{\min\{c_0,1\}} \left(\frac{\gamma(c_0 - c)}{2(1-\phi)}\right)\left(\frac{c + c_0 + 2w}{2(1-\phi)}\right) dc}{\int_0^{\min\{c_0,1\}} \left(\frac{\gamma(c_0 - c)}{2(1-\phi)}\right) dc} \\ \iff 0 &= \begin{cases} \dfrac{\gamma(c_0 + w) + \phi - 1}{1-\phi} - \dfrac{(2c_0 + 3w)(\gamma - \beta)}{3(1-\phi)} & \text{, if } c_0 \leq 1, \\ \dfrac{\gamma(c_0 + w) + \phi - 1}{1-\phi} - \dfrac{(3c_0^2 + 6c_0 w - 3w - 1)(\gamma - \beta)}{3(2c_0 - 1)(1-\phi)} & \text{, if } c_0 > 1. \end{cases} \end{aligned} \tag{15}$$

The right-hand side of (15) is continuous at $c_0 = 1$. It is also monotonically increasing in c_0:

$$\frac{\partial}{\partial c_0}\left(\frac{\gamma(c_0+w)+\phi-1}{(1-\phi)} - \frac{(2c_0+3w)(\gamma-\beta)}{3(1-\phi)}\right) = \frac{2\beta+\gamma}{3(1-\phi)} > 0\ \forall c_0 \leq 1,$$

$$\frac{\partial}{\partial c_0}\left(\frac{\gamma(c_0+w)+\phi-1}{1-\phi} - \frac{(3c_0^2+6c_0w-3w-1)(\gamma-\beta)}{3(2c_0-1)(1-\phi)}\right) = \frac{\gamma+\beta(6(c_0-1)c_0+2)+6\gamma(c_0-1)c_0}{3(2c_0-1)^2(1-\phi)} > 0\ \forall c_0 > 1.$$

As a result, the right-hand side of (15) can only cross zero once, yielding a unique c_0.

The unique candidate c_0 in the domain $c_0 \leq 1$ is

$$c_0 = \frac{3(1-\phi-\beta w)}{2\beta+\gamma}. \tag{16}$$

From (14) and (16), for this candidate c_0, there exists a unique equilibrium average price,

$$\overline{p} = \frac{2(1-\phi)+\gamma w}{(1-\phi)(2\beta+\gamma)}. \tag{17}$$

$\square$

Proof of Proposition 3. The uniqueness of an average price that satisfies consistent expectations equilibrium is guaranteed by Proposition 2. Due to equivalency of the expected average price, $\overline{p}_e$, and realized average price, $\overline{p}$, we do not make the distinction between the two throughout the proofs.

In decentralized pricing with commission fees, conditional on participation, an agent with cost c setting a price p earns

$$\pi_c(p,\overline{p}) = (1-\beta\overline{p}+\gamma(\overline{p}-p))((1-\phi)p-c).$$

Agent's profit depends on individual price, p, and also the average market price, $\overline{p}$, which is characterized in the equilibrium. Since each agent is small, the price set by an individual agent does not influence the average market price.

Each agent chooses the price that maximizes own earnings. From (10),

$$p^*(c,\overline{p}) = \frac{1}{2\gamma}\left(1+(\gamma-\beta)\overline{p}+\frac{\gamma c}{1-\phi}\right).$$

The agent earns

$$\pi_c(p^*(c,\overline{p}),\overline{p}) = \frac{(1-\phi-\overline{p}(1-\phi)(\beta-\gamma)-\gamma c)^2}{4\gamma(1-\phi)}.$$

Agent earnings are decreasing in cost, c, and therefore there exists a threshold cost, c_h, for which agent with cost c participates if and only if $c \leq c_h$. Define c_0 to be the lowest cost that yields zero profit:

$$\pi_{c_0}(p^*(c_0,\overline{p}),\overline{p}) = 0 \implies c_0 = c_0(\phi) = \frac{(1-\phi)(1+(\gamma-\beta)\overline{p})}{\gamma},$$

Then,

$$c_h = c_h(\phi) = \min\{1, c_0\}.$$

If $c_0 \leq 1$, the average market price is uniquely defined by (17):

$$\overline{p} = \frac{2}{2\beta+\gamma}.$$

The platform's profit-maximization problem is:

$$\begin{aligned}\max_{\phi} \quad \Pi^{\mathcal{D}}(\phi) &= \phi\overline{p} \int_0^{c_h(\phi)} q(p^*(c,\overline{p}),\overline{p})\, dc \\ &= \phi\overline{p} \int_0^{c_0(\phi)} q(p^*(c,\overline{p}),\overline{p})\, dc \\ &= \frac{9\gamma(1-\phi)\phi}{2(2\beta+\gamma)^3}.\end{aligned}$$

The objective function is concave in ϕ and is maximized at $\phi = 1/2$. It yields a profit of

$$\Pi^{\mathcal{D}} = \frac{9\gamma(1-\phi)\phi}{2(2\beta+\gamma)^3} = \frac{9}{8}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right).$$

This is a feasible solution because

$$\begin{aligned}c_0 &= \frac{(1-\phi)(1+(\gamma-\beta)\overline{p})}{\gamma} \\ &= \frac{3}{4\beta+2\gamma} < 1,\end{aligned}$$

which follows from the $\beta > 1$ restriction discussed in Section 3.

If $c_0 > 1$, the average price is determined by (15). The platform's profit is

$$\begin{aligned}\Pi^{\mathcal{D}}(\phi) &= \phi\overline{p} \int_0^{c_h(\phi)} q(p^*(c,\overline{p}),\overline{p})\, dc \\ &= \phi\overline{p} \int_0^{1} q(p^*(c,\overline{p}),\overline{p})\, dc \\ &= \phi\left(\frac{(1+(\gamma-\beta)\overline{p})^2}{4\gamma} - \frac{\gamma}{12(1-\phi)^2}\right) \\ &= \frac{\gamma\left(3c_0^2-1\right)\phi}{12(1-\phi)^2}\end{aligned} \tag{18}$$

where the last equality follows by plugging in (14). We will show that the optimal solution does not reside in any set of ϕ, $c_0 > 1$ that satisfies (15). As a first step, apply implicit function theorem on (15) to get

$$\frac{\partial c_0}{\partial \phi} = -\frac{3(2c_0-1)^2}{\gamma+\beta(6(c_0-1)c_0+2)+6\gamma(c_0-1)c_0} < 0\ \forall c_0 > 1.$$

Using the partial derivative, take the total derivative of (18) with respect to ϕ:

$$\begin{aligned}\frac{d\Pi^{\mathcal{D}}(\phi)}{d\phi} = \frac{\gamma}{12(\phi-1)^3(\gamma+\beta(6(c_0-1)c_0+2)+6\gamma(c_0-1)c_0)}\Big(&\gamma(6(1-c_0)c_0-1)\left(3c_0^2-1\right)(\phi+1) \\ &-2\beta\left(9c_0^4-9c_0^3+3c_0-1\right)(\phi+1) - 18c_0(1-2c_0)^2(\phi-1)\phi\Big).\end{aligned}$$

The right-hand side is positive for all $c_0 > 1$ and $1 > \phi > 0$ whenever $\beta > 1$, and therefore

$$\frac{d\Pi^{\mathcal{D}}(\phi)}{d\phi} > 0.$$

This means, in any solution with $c_0 > 1$, the profits are increasing in ϕ. Together with $\partial\phi/\partial c_0 < 0$, this implies that we cannot have an optimal with $c_0 > 1$. With any candidate optimal ϕ satisfying $c_0 > 1$, you could improve the solution by increasing ϕ, which lowers c_0, creating a contradiction. Hence, the optimal solution resides within the case $c_0 \leq 1$.

The highest cost that participates is

$$c_h = \frac{(1-\phi)(1+(\gamma-\beta)\overline{p})}{\gamma} = \frac{3}{2(2\beta+\gamma)}.$$

Total quantity of customers served in the market is

$$\begin{aligned} Q &= \int_0^{c_h(\phi)} q(p^*(c,\overline{p}),\overline{p})\, dc \\ &= \frac{(1-\phi)(1+(\gamma-\beta)\overline{p})^2}{4\gamma} \\ &= \frac{9}{8}\left(\frac{\gamma}{(2\beta+\gamma)^2}\right). \end{aligned}$$

Agents' total earnings is

$$\begin{aligned} \pi^{\mathcal{D}} &= \int_0^{c_h(\phi)} q(p^*(c,\overline{p}),\overline{p})((1-\phi)p^*(c,\overline{p}) - c)\, dc \\ &= \frac{9\gamma(1-\phi)^2}{4(2\beta+\gamma)^3} \\ &= \frac{9}{16}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right). \end{aligned}$$

□

Proof of Proposition 4. The ratio of platform's profits under decentralized pricing and centralized pricing is:

$$\frac{\Pi^{\mathcal{D}}}{\Pi^{\mathcal{C}}}(\gamma,\beta) = \frac{\frac{9\gamma}{8(2\beta+\gamma)^3}}{\frac{1}{27\beta^2}} = \frac{243\beta^2\gamma}{8(2\beta+\gamma)^3}.$$

The platform prefers decentralized pricing if

$$\begin{aligned} \frac{243\beta^2\gamma}{8(2\beta+\gamma)^3} > 1 &\iff -64\beta^3 + 147\beta^2\gamma - 48\beta\gamma^2 - 8\gamma^3 > 0 \\ &\iff -64\left(\frac{\beta}{\gamma}\right)^3 + 147\left(\frac{\beta}{\gamma}\right)^2 - 48\left(\frac{\beta}{\gamma}\right) - 8 > 0 \\ &\iff 0.539\beta < \gamma < 1.785\beta. \end{aligned}$$

□

Proof of Corollary 1. With both centralized pricing and decentralized pricing, agents' total earnings is equal to half of the platform's:

$$\frac{\pi^{\mathcal{C}}}{\Pi^{\mathcal{C}}} = \frac{\dfrac{1}{27\beta^2}}{\dfrac{1}{54\beta^2}} = \frac{1}{2} = \frac{\dfrac{9}{8}\left(\dfrac{\gamma}{(2\beta+\gamma)^3}\right)}{\dfrac{9}{16}\left(\dfrac{\gamma}{(2\beta+\gamma)^3}\right)} = \frac{\pi^{\mathcal{D}}}{\Pi^{\mathcal{D}}}.$$

Therefore, agents' aggregate welfare is higher with decentralized pricing if and only if platform's profit is higher with decentralized pricing. By Proposition 4, this is true if

$$0.539\beta < \gamma < 1.785\beta.$$

To show that not all agents are better off with decentralization, let $\gamma = 3\beta/2$, $\beta = 2$ and consider the agent with cost $c = 1/10$. Numerical evaluation determines the agent earns $1/45 \approx 0.022$ with centralized pricing, whereas the agent earns less, $24/1225 \approx 0.020$, with decentralized pricing.

□

Proof of Proposition 5 According to the Revelation Principle (Myerson 1981) the set of optimal mechanisms can be found within the set of truth-inducing mechanisms. In such a mechanism, the platform posts a menu of costs, and prices and fees associated with each cost. Each agent reports a cost and is assigned the price and fee associated with that cost, and each agent (i) earns non-negative profits (Individual Rationality), and (ii) prefers to report own cost truthfully given that all other agents do the same (Incentive Compatibility). Following this structure, let $p(c)$ be the price the platform assigns to agent c and let $F(c)$ be the fee collected. Note that $F(c)$ is the total fee and not per-unit, $F(c) = q(p(c), \overline{p})f(c)$. Let $\pi_c(\tilde{c})$ be an agent's earnings with cost c when the agent reports cost $\tilde{c}$:

$$\pi_c(\tilde{c}) = u(c, p(\tilde{c})) - F(\tilde{c})$$

where

$$u(c, p(\tilde{c})) = (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))\,(p(\tilde{c}) - c).$$

The Individual Rationality (IR) and the Incentive Compatibility (IC) constraints are

$$\pi_c(c) \geq 0,$$

$$\pi_c(c) \geq \pi_c(\tilde{c}),$$

respectively for all $c \in [0, 1]$, $\tilde{c} \in [0, 1]$.

Because of the IC constraints, agent earnings are strictly decreasing in cost:

$$\begin{aligned}\pi_c(c) \geq \pi_c(\tilde{c}) &= (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))\,(p(\tilde{c}) - c) - F(\tilde{c}) \\ &> (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))\,(p(\tilde{c}) - \tilde{c}) - F(\tilde{c}) = \pi_{\tilde{c}}(\tilde{c})\end{aligned}$$

for all $\tilde{c} > c$. This also implies that the agent with the highest cost, $c = 1$, earns zero under the optimal truth-inducing mechanism: if the highest cost agent were to earn a strictly positive amount, then the platform could uniformly increase the fee, $F(\tilde{c})$, for all participating agents, thereby increasing its profit.

We can re-formulate an agent's earnings as

$$\pi_c(\tilde{c}) = \pi_{\tilde{c}}(\tilde{c}) - u(\tilde{c}, p(\tilde{c})) + u(c, p(\tilde{c})).$$

For a pair of agents with costs c, $\tilde{c}$, IC constraints imply

$$\begin{aligned}\pi_c(c) \geq \pi_c(\tilde{c}) &= \pi_{\tilde{c}}(\tilde{c}) - u(\tilde{c}, p(\tilde{c})) + u(c, p(\tilde{c})), \\ \pi_{\tilde{c}}(\tilde{c}) \geq \pi_{\tilde{c}}(c) &= \pi_c(c) - u(c, p(c)) + u(\tilde{c}, p(c)).\end{aligned}$$

These inequalities can be combined:

$$\begin{aligned}& u(\tilde{c}, p(c)) - u(c, p(c)) \leq \pi_{\tilde{c}}(\tilde{c}) - \pi_c(c) \leq u(\tilde{c}, p(\tilde{c})) - u(c, p(\tilde{c})) \\ \iff & \int_c^{\tilde{c}} \frac{\partial u(c_k, p(c))}{\partial c_k}\, dc_k \leq \pi_{\tilde{c}}(\tilde{c}) - \pi_c(c) \leq \int_c^{\tilde{c}} \frac{\partial u(c_k, p(\tilde{c}))}{\partial c_k}\, dc_k.\end{aligned} \tag{19}$$

This inequality has two important consequences. First, ignoring the middle term, Equation (19) implies

$$\begin{aligned}& \int_c^{\tilde{c}} \frac{\partial u(c_k, p(c))}{\partial c_k}\, dc_k \leq \int_c^{\tilde{c}} \frac{\partial u(c_k, p(\tilde{c}))}{\partial c_k}\, dc_k, \ \forall c \in [0,1], \forall \tilde{c} \in [0,1] \\ \iff & -\int_c^{\tilde{c}} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))\, dc_k \leq -\int_c^{\tilde{c}} (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))\, dc_k, \ \forall c \in [0,1], \forall \tilde{c} \in [0,1] \\ \iff & (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))\,(c - \tilde{c}) \leq (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))\,(c - \tilde{c}), \ \forall c \in [0,1], \forall \tilde{c} \in [0,1] \\ \iff & 0 \leq (p(c) - p(\tilde{c}))(c - \tilde{c}), \ \forall c \in [0,1], \forall \tilde{c} \in [0,1] \\ \iff & p'(c) \geq 0, \ \forall c \in [0,1].\end{aligned} \tag{20}$$

Second, because $u(\tilde{c}, p(c))$ is a continuous function of $\tilde{c}$, this inequality implies that $\pi_{\tilde{c}}(\tilde{c})$ is also continuous with respect to the Lebesgue measure and thus, almost everywhere differentiable. Furthermore, its derivative is

$$\frac{d\pi_c(c)}{dc} = \frac{\partial u(c, p(c))}{\partial c}.$$

Using the fundamental theorem of calculus:

$$\pi_{\tilde{c}}(\tilde{c}) = \pi_c(c) + \int_c^{\tilde{c}} \frac{d\pi_{c_k}(c_k)}{dc_k}\, dc_k = \pi_c(c) + \int_c^{\tilde{c}} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k.$$

Because the highest cost agent earns zero, setting $\tilde{c} = 1$, the equation simplifies to

$$\pi_c(c) = -\int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k,$$

which means

$$F(c) = u(c, p(c)) + \int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k. \tag{21}$$

Our analysis so far established that (20) and (21) are necessary conditions for IR and IC. Now, we will show that they are also sufficient. By establishing an equivalence between the two sets of constraints, we will be able to replace the IR and IC constraints in the platform's optimal mechanism problem with Equations (20) and (21) and therefore convert the problem into a tractable form.

It is straightforward to show that (21) is sufficient to imply IR:

$$\begin{aligned}
\pi_c(c) &= u(c, p(c)) - F(c) \\
&= u(c, p(c)) - u(c, p(c)) - \int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \\
&= -\int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \\
&= \int_c^1 (1 - \beta\overline{p} + \gamma(\overline{p} - p(c_k)))\, dc_k \geq 0\ \forall c \in [0, 1].
\end{aligned}$$

Similarly for IC, we have

$$\begin{aligned}
&\pi_c(\tilde{c}) - \pi_c(c) = u(c, p(\tilde{c})) - F(\tilde{c}) - u(c, p(c)) + F(c) \\
=&u(c, p(\tilde{c})) - u(\tilde{c}, p(\tilde{c})) - \int\limits_{\tilde{c}}^{1} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k - u(c, p(c)) + u(c, p(c)) + \int\limits_{c}^{1} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \\
=&u(c, p(\tilde{c})) - u(\tilde{c}, p(\tilde{c})) - \int\limits_{\tilde{c}}^{c} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \\
=&\int\limits_{\tilde{c}}^{c} \frac{\partial u(c_k, p(\tilde{c}))}{\partial c_k}\, dc_k - \int\limits_{\tilde{c}}^{c} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \\
=&\int\limits_{\tilde{c}}^{c} (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))\, dc_k - \int\limits_{\tilde{c}}^{c} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c_k)))\, dc_k \\
=&\gamma \int\limits_{\tilde{c}}^{c} (p(c_k) - p(\tilde{c}))\, dc_k \geq 0,\ \forall c \in [0,1], \tilde{c} \in [0,1].
\end{aligned}$$

Now, we construct the platform's problem. The platform chooses $p(c)$ and $F(c)$ to maximize total profits subject to IR and IC constraints, and the natural restriction that the demand of an agent cannot be negative:

$$\begin{aligned}
\max_{p(c),F(c)} \quad & \int\limits_{0}^{1} F(c)\, dc \\
\text{s.t.} \quad & \pi_c(c) \geq \pi_c(\tilde{c}), \forall c \in [0,1], \forall \tilde{c} \in [0,1] \\
& \pi_c(c) \geq 0, \forall c \in [0,1] \\
& 1 - \beta\overline{p} + \gamma(\overline{p} - p(c)) \geq 0, \forall c \in [0,1] \\
& \text{Eq. (2).}
\end{aligned} \tag{22}$$

Using the equivalence we found earlier, we replace IR and IC constraints with Equations (20) and

(21):

$$
\begin{aligned}
\max_{p(c),F(c)} \quad & \int_0^1 F(c)\, dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in [0,1] \\
& F(c) = u(c, p(c)) + \int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k, \forall c \in [0,1] \\
& 1 - \beta\overline{p} + \gamma(\overline{p} - p(c)) \geq 0, \forall c \in [0,1] \\
& \text{Eq. (2)} \\
= \max_{p(c)} \quad & \int_0^1 \left(u(c, p(c)) + \int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \right) dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in [0,1] \\
& 1 - \beta\overline{p} + \gamma(\overline{p} - p(c)) \geq 0, \forall c \in [0,1] \\
& \text{Eq. (2).}
\end{aligned}
$$

Using integration by parts,

$$
\begin{aligned}
\int_0^1 \left(\int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \right) dc &= \left[\left(\int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \right) c \right]_0^1 - \int_0^1 \left(-\frac{\partial u(c, p(c))}{\partial c} \right) c\, dc \\
&= \int_0^1 \frac{\partial u(c, p(c))}{\partial c} c\, dc.
\end{aligned} \tag{23}
$$

The platform's problem converts to

$$
\begin{aligned}
\max_{p(c)} \quad & \int_0^1 \left(u(c, p(c)) + \frac{\partial u(c, p(c))}{\partial c} c \right) dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in [0,1] \\
& 1 - \beta\overline{p} + \gamma(\overline{p} - p(c)) \geq 0, \forall c \in [0,1] \\
& \text{Eq. (2)} \\
= \max_{p(c)} \quad & \int_0^1 \Big((1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))(p(c) - c) - (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))c \Big)\, dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in [0,1] \\
& 1 - \beta\overline{p} + \gamma(\overline{p} - p(c)) \geq 0, \forall c \in [0,1] \\
& \text{Eq. (2)} \\
= \max_{p(c)} \quad & \int_0^1 (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))(p(c) - 2c)\, dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in [0,1] \\
& 1 - \beta\overline{p} + \gamma(\overline{p} - p(c)) \geq 0, \forall c \in [0,1] \\
& \text{Eq. (2).}
\end{aligned} \tag{24}
$$

We re-formulate this as a quantity-choice problem. Let $q(c)$ be the demand platform assigns to agent with cost c. By Equation (1), we have one-to-one equivalence between price and quantity:

$$p(c) = \frac{1 + (\gamma - \beta)\overline{p} - q(c)}{\gamma}.$$

The average price in Equation (2) is

$$\begin{aligned}
\overline{p} &= \frac{\int_0^1 q(c) \left(\frac{1 + (\gamma - \beta)\overline{p} - q(c)}{\gamma} \right) dc}{\int_0^1 q(c)\, dc} \\
&= \frac{\int_0^1 \frac{q(c)\,(1 - q(c))}{\gamma}\, dc}{\int_0^1 q(c)\, dc} + \overline{p}\left(1 - \frac{\beta}{\gamma}\right),
\end{aligned}$$

which simplifies to

$$\overline{p} = \frac{\int_0^1 q(c)\,(1 - q(c))\; dc}{\beta \int_0^1 q(c)\, dc}. \tag{25}$$

Platform's optimal quantity-choice problem is

$$\begin{aligned}
&\max_{q(c)} \int_0^1 q(c) \left(\frac{1 + (\gamma - \beta)\overline{p} - q(c)}{\gamma} - 2c \right)\; dc \\
&\text{s.t. } q'(c) \le 0, \forall c \in [0, 1] \\
&\qquad q(c) \ge 0, \forall c \in [0, 1] \\
&\qquad \text{Eq. (25).}
\end{aligned}$$

We can further simplify the objective function. Notice that Equation (25) implies:

$$\overline{p} \int_0^1 q(c)\, dc = \frac{1}{\beta} \int_0^1 q(c)\,(1 - q(c))\; dc.$$

Using this relationship,

$$
\begin{aligned}
\Pi &= \int_0^1 q(c)\left(\frac{1+(\gamma-\beta)\overline{p}-q(c)}{\gamma}-2c\right)\,dc \\
&= \int_0^1 q(c)\left(\frac{1-q(c)}{\gamma}-2c\right)\,dc + \left(\frac{\gamma-\beta}{\gamma}\right)\overline{p}\int_0^1 q(c)\,dc \\
&= \int_0^1 q(c)\left(\frac{1-q(c)}{\gamma}-2c\right)\,dc + \left(\frac{\gamma-\beta}{\gamma}\right)\frac{1}{\beta}\int_0^1 q(c)\left(1-q(c)\right)\,dc \\
&= \int_0^1 \left(q(c)\left(\frac{1-q(c)}{\gamma}-2c\right) + \left(\frac{\gamma-\beta}{\gamma}\right)\frac{1}{\beta}q(c)\left(1-q(c)\right)\right)\,dc \\
&= \int_0^1 q(c)\left(\frac{1-q(c)}{\beta}-2c\right)\,dc.
\end{aligned}
\tag{26}
$$

The problem terms no longer depend on the average price:

$$
\begin{aligned}
&\max_{q(c)} \int_0^1 q(c)\left(\frac{1-q(c)}{\beta}-2c\right)\,dc \\
&\text{s.t. } q'(c) \leq 0, \forall c \in [0,1] \\
&\qquad q(c) \geq 0, \forall c \in [0,1].
\end{aligned}
\tag{27}
$$

Relaxing the first constraint, we can decompose the problem into individual sub-problems for all agents:

$$
\begin{aligned}
&\max_{q(c)} q(c)\left(\frac{1-q(c)}{\beta}-2c\right)\,dc \\
&\text{s.t. } q(c) \geq 0.
\end{aligned}
$$

This is the maximization of a simple quadratic function with a linear constraint. The optimal quantity is:

$$
q^*(c) = \max\left\{0, \frac{1}{2}-\beta c\right\}.
$$

The highest cost agent with non-zero quantity is

$$
c_h = \min\left\{1, \frac{1}{2\beta}\right\} = \frac{1}{2\beta}.
$$

The second equality follows from the $\beta > 1$ restriction discussed in Section 3. This solution also satisfies:

$$
q^{*\prime}(c) \leq 0.
$$

Therefore, the solution our relaxed problem is also optimal for the platform's optimal mechanism.

The average market price is:

$$\begin{aligned}\overline{p} &= \frac{\int_0^1 q^*(c)\,(1-q^*(c))\; dc}{\beta \int_0^1 q^*(c)\, dc} \\ &= \frac{2}{3\beta}.\end{aligned}$$

Other equilibrium characteristics are as follows:

$$\begin{aligned}p^*(c) &= \frac{1+(\gamma-\beta)\overline{p}-q^*(c)}{\gamma} \\ &= c^2\left(\frac{\beta(\gamma-2\beta)}{2\gamma}\right)+\frac{2c}{3}\left(\frac{\beta}{\gamma}-1\right)+\frac{1}{24}\left(\frac{5}{\beta}-\frac{2}{\gamma}\right).\end{aligned}$$

The per-unit fee collected is

$$\begin{aligned}f^*(c) = \frac{F^*(c)}{q^*(c)} &= \frac{1}{12}\left(\frac{5}{\beta}-\frac{2}{\gamma}\right)+\left(\frac{\beta}{\gamma}-\frac{1}{2}\right)c \\ &= \left(1-\frac{\gamma}{2\beta}\right)\left(\frac{2}{3\beta}-\frac{1}{6\gamma}+\frac{c\beta}{\gamma}\right)+\frac{\gamma-\beta}{3\beta^2} \\ &= \left(1-\frac{\gamma}{2\beta}\right)p^*(c)+\frac{\gamma-\beta}{3\beta^2}.\end{aligned}$$

Because we can map the fee structure of the optimal mechanism to one that is linear in price, Proposition 2 applies also to the optimal mechanism. Therefore, with the optimal mechanism, there exists a unique equilibrium average price.

The platform earns

$$\begin{aligned}\Pi &= \int_0^1 q^*(c)\left(\frac{1-q^*(c)}{\beta}-2c\right)\; dc \\ &= \frac{1}{24\beta^2}.\end{aligned}$$

Total quantity of customers served in the market is

$$\begin{aligned}Q &= \int_0^1 q^*(c)\; dc \\ &= \frac{1}{8\beta}.\end{aligned}$$

Agents' total earnings is

$$\begin{aligned}\pi &= \int_0^1 \pi_c(c)\; dc \\ &= \frac{1}{48\beta^2}.\end{aligned}$$

□

Proof of Proposition 6. With the optimal mechanism, we have

$$\begin{aligned} F^*(c) &= c^2\left(\frac{\beta(\gamma-2\beta)}{2\gamma}\right)+\frac{2c}{3}\left(\frac{\beta}{\gamma}-1\right)+\frac{1}{24}\left(\frac{5}{\beta}-\frac{2}{\gamma}\right) \\ &= q^*(c)\left(\left((1-\frac{\gamma}{2\beta}\right)p^*(c)+\frac{\gamma-\beta}{3\beta^2}\right). \end{aligned}$$

There is a unique candidate commission-plus fee structure that may yield the same equilibrium outcome as the optimal mechanism. To show that this structure indeed replicates the optimal mechanism, let the platform set its terms

$$\phi = 1-\frac{\gamma}{2\beta},\ w=\frac{\gamma-\beta}{3\beta^2},$$

and assume agents expect an average price of

$$\overline{p}=\frac{2}{3\beta}.$$

By Equation (10), an agent with cost c posts a price under commission-plus

$$\begin{aligned} p^*(c,\overline{p}) &= \frac{1}{2}\left(\frac{1-(\beta-\gamma)\overline{p}}{\gamma}+\frac{c+w}{1-\phi}\right) \\ &= \frac{2}{3\beta}-\frac{1}{6\gamma}+\frac{c\beta}{\gamma}. \end{aligned}$$

The price chosen by any agent is exactly equivalent to the one in the optimal mechanism. Same is true for the fees paid. Because prices and fees are the only decision variables in the setting, then commission-plus with the given terms replicates the optimal mechanism.

□

Proof of Proposition 7. With the optimal mechanism, we have

$$\begin{aligned} F^*(c) &= c^2\left(\frac{\beta(\gamma-2\beta)}{2\gamma}\right)+\frac{2c}{3}\left(\frac{\beta}{\gamma}-1\right)+\frac{1}{24}\left(\frac{5}{\beta}-\frac{2}{\gamma}\right) \\ &= q^*(c)\left(\left(\frac{1}{2\beta}-\frac{1}{\gamma}\right)q^*(c)+\frac{1}{6\beta}+\frac{1}{3\gamma}\right). \end{aligned}$$

There is a unique candidate quantity pricing fee structure that may yield the same equilibrium outcome as the optimal mechanism. To show that this structure indeed replicates the optimal mechanism, let the platform sets its terms

$$\phi=\frac{1}{2\beta}-\frac{1}{\gamma},\ w=\frac{1}{6\beta}+\frac{1}{3\gamma}.$$

Conditional on participation, an agent with cost c setting a price p earns

$$\pi_c(p,\overline{p})=(1-\beta\overline{p}+\gamma(\overline{p}-p))(p-\phi(1-\beta\overline{p}+\gamma(\overline{p}-p))-c-w).$$

Because earnings are strictly concave in price, there exists a unique price, $p^*(c, \overline{p})$, that maximizes the agent's earnings,

$$p^*(c, \overline{p}) = \frac{1 - \overline{p}(\beta - \gamma)(1 + 2\gamma\phi) + \gamma(c + w + 2\phi)}{2\gamma(1 + \gamma\phi)}$$
$$= \frac{2}{3\beta} - \frac{1}{6\gamma} + \frac{c\beta}{\gamma}.$$

The price chosen by any agent is exactly equivalent to the one in the optimal mechanism. Same is true for the fees paid. Because prices and fees are the only decision variables in the setting, then quantity pricing with the given terms replicates the optimal mechanism.

□

Proof of Proposition 8. We will re-solve centralized pricing, decentralized pricing, optimal mechanism and commission-plus models under the assumption that agents have a maximum capacity to serve k demand.

Centralized pricing: Without capacity constraints, the number of customers an agent serves under centralized pricing is

$$q(\overline{p}) = 1 - \beta\overline{p} = \frac{1}{3}.$$

With capacity constraints, the capacity will be binding if and only if $k \leq \frac{1}{3}$. When capacity is binding, if platform sets a price $\overline{p}$, the profit agent with cost c earns is

$$\pi_c(\overline{p}) = k((1 - \phi)\overline{p} - c).$$

Agents participate as long as they earn positive profits. The highest cost that participates, c_h, earns zero

$$\pi_{c_h}(\overline{p}) = 0 \implies c_h = c_h(\overline{p}, \phi) = (1 - \phi)\overline{p}.$$

The platform's profit is:

$$\Pi^{\mathcal{C}}(\overline{p}, \phi) = \int_0^{c_h(\overline{p},\phi)} k\phi\overline{p}\, dc$$
$$= k\phi(1 - \phi)\overline{p}^2.$$

The objective is increasing monotonically in $\overline{p}$. It is optimal for platform to increase the price as long as there is excess capacity. In the optimal, quantity served is equal to the capacity:

$$k = q(\overline{p}) = 1 - \beta\overline{p} \implies \overline{p} = \frac{1 - k}{\beta}.$$

The objective is also concave in ϕ and is maximized at $\phi = 1/2$.

Plugging in the solution, the platform's optimal profit is

$$\Pi^{\mathcal{C}} = \frac{k(1-k)^2}{4\beta^2}.$$

<u>Decentralized pricing with commission</u>: Without capacity constraints, the maximum number of customers an agent serves under decentralized pricing with just commission is

$$q(p^*(0,\overline{p}),\overline{p}) = 1 - \beta\overline{p} + \gamma(\overline{p} - p^*(0,\overline{p})) = \frac{3\gamma}{4\beta + 2\gamma}.$$

With capacity constraints, the capacity will be binding if and only if $k \leq \frac{3\gamma}{4\beta+2\gamma}$. The agents need to incorporate the capacity constraint in their pricing decision. Profit earned by agent with cost c and price p conditional on participation is

$$\pi_c(p,\overline{p}) = \min\{1 - \beta\overline{p} + \gamma(\overline{p} - p), k\}((1-\phi)p - c).$$

Agent sets the price that maximizes own profits:

$$\max_p \min\{1 - \beta\overline{p} + \gamma(\overline{p} - p), k\}((1-\phi)p - c).$$

Without capacity constraints, an agent's optimal price is characterized by Equation (10),

$$p^*(c,\overline{p}) = \frac{1}{2\gamma}\left(1 + (\gamma - \beta)\overline{p} + \frac{c\gamma}{1-\phi}\right),$$

which gives quantity

$$q(p^*(c,\overline{p}),\overline{p}) = \frac{1}{2}\left(1 + (\gamma - \beta)\overline{p} - \frac{c\gamma}{1-\phi}\right).$$

Since quantity served is decreasing in cost, there exists a threshold, $\tilde{c}$, where only the agents costs $c \leq \tilde{c}$ are capacity constrained. The threshold agent has exactly the quantity k:

$$q(p^*(\tilde{c},\overline{p}),\overline{p}) = k \implies \tilde{c} = \tilde{c}(\phi) = \frac{(1-\phi)(1 + (\gamma-\beta)\overline{p} - 2k)}{\gamma}.$$

If an agent is capacity constrained ($k \leq q(p^*(c,\overline{p}),\overline{p})$), any decrease in price further than the one that matches capacity and demand decreases the agent's margin, but not increase quantity served. Hence, an agent never sets a price lower than the amount that makes capacity binding. The optimal price of an agent with cost $c < \tilde{c}$ is:

$$p^*(\tilde{c},\overline{p}) = \frac{1 + (\gamma - \beta)\overline{p} - k}{\gamma}.$$

The highest cost agent that participates has 0 demand:

$$q(p^*(c_h,\overline{p}),\overline{p}) = 0 \implies c_h = c_h(\phi) = \frac{(1-\phi)(1 + (\gamma-\beta)\overline{p})}{\gamma}.$$

Agents' equilibrium prices are

$$p^*(c,\overline{p}) = \begin{cases} \dfrac{1+(\gamma-\beta)\overline{p}-k}{\gamma}, & c \le \tilde{c}, \\ \dfrac{1}{2}\left(\dfrac{1+(\gamma-\beta)\overline{p}}{\gamma} + \dfrac{c}{1-\phi}\right), & \tilde{c} < c \le c_h. \end{cases}$$

Agents with cost higher than c_h cannot profitably participate.

The average market price is defined in the equilibrium as a weighted average of all prices set in the market. The average price that occurs by the agent's optimal decisions is consistent with their expectation of the average price. By Equation (2):

$$\begin{aligned} \overline{p} &= \frac{\int_0^{c_h} q(p^*(c,\overline{p}),\overline{p})p^*(c,\overline{p})\, dc}{\int_0^{c_h} q(p^*(c,\overline{p}),\overline{p})\, dc} \\ &= \frac{\int_0^{\tilde{c}} kp^*(\tilde{c},\overline{p})\, dc + \int_{\tilde{c}}^{c_h} \big(1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p}))\big)p^*(c,\overline{p})\, dc}{\int_0^{\tilde{c}} k\, dc + \int_{\tilde{c}}^{c_h} \big(1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p}))\big)\, dc} \\ &= \frac{3(\overline{p}(\gamma-\beta)+1)^2+6k(\overline{p}(\beta-\gamma)-1)+4k^2}{3\gamma(\overline{p}(\gamma-\beta)-k+1)}. \end{aligned}$$

There is a unique feasible equilibrium average price:

$$\overline{p} = \frac{\sqrt{-12\beta^2k^2+12\beta\gamma k^2+9\gamma^2(k-1)^2}+6\beta(k-1)-3\gamma(k-1)}{6\beta(\beta-\gamma)}.$$

Platform's profit maximization problem is:

$$\begin{aligned} \max_{\phi} \quad \Pi^{\mathcal{D}} &= \phi\overline{p}\left(\int_0^{\tilde{c}} (1-\beta\overline{p}+\gamma(\overline{p}-p^*(\tilde{c},\overline{p})))\, dc + \int_{\tilde{c}}^{c_h} (1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p})))\, dc\right) \\ &= \frac{k\left(-k\sqrt{-12\beta^2k^2+12\beta\gamma k^2+9\gamma^2(k-1)^2}+\sqrt{-12\beta^2k^2+12\beta\gamma k^2+9\gamma^2(k-1)^2}+2\beta k^2+3\gamma(k-1)^2\right)}{6\beta^2\gamma}(1-\phi)\phi. \end{aligned}$$

Platform's profit function is concave in ϕ and is maximized at $\phi = 1/2$. Platform's profit is

$$\Pi^{\mathcal{D}} = \frac{k\left(-k\sqrt{-12\beta^2k^2+12\beta\gamma k^2+9\gamma^2(k-1)^2}+\sqrt{-12\beta^2k^2+12\beta\gamma k^2+9\gamma^2(k-1)^2}+2\beta k^2+3\gamma(k-1)^2\right)}{24\beta^2\gamma}.$$

Optimal mechanism: Under the optimal mechanism, the highest number of customers an agent serves is $q^*(0) = 1/2$. Therefore, the capacity is binding if and only if $k \le \frac{1}{2}$.

Our proof for platform's optimal mechanism for the demand unconstrained model in Proposition 5 is applicable here up until we reach Equation (27). In the presence of demand constraints, platform's optimal mechanism problem has an additional constraint:

$$\begin{aligned}
&\max_{q(c)} \int_0^1 q(c)\left(\frac{1-q(c)}{\beta}-2c\right)\,dc \\
&\text{s.t. } q'(c) \le 0, \forall c \in [0,1] \\
&\quad\;\; q(c) \ge 0, \forall c \in [0,1] \\
&\quad\;\; q(c) \le k, \forall c \in [0,1].
\end{aligned}$$

Relaxing the first constraint, we can decompose this into individual sub-problems for all agents:

$$\begin{aligned}
&\max_{q(c)} q(c)\left(\frac{1-q(c)}{\beta}-2c\right)\,dc \\
&\text{s.t. } q(c) \ge 0 \\
&\quad\;\; q(c) \le k.
\end{aligned}$$

This is the maximization of a simple quadratic function with two linear constraints. The optimal quantity is:

$$q^*(c) = \min\left\{k, \max\left\{0, \frac{1}{2}-\beta c\right\}\right\}.$$

The highest cost agent with non-zero quantity is

$$c_h = \min\left\{1, \frac{1}{2\beta}\right\} = \frac{1}{2\beta}.$$

The last equality follows from the $\beta > 1$ restriction discussed in Section 3.

This solution also satisfies:

$$q^{*\prime}(c) \le 0.$$

Therefore, the solution to the relaxed problem is also optimal for the platform's optimal mechanism problem.

The average market price is:

$$\begin{aligned}
\overline{p} &= \frac{\int_0^1 q^*(c)\left(1-q^*(c)\right)\,dc}{\beta\int_0^1 q^*(c)\,dc} \\
&= \frac{4k^2-6k+3}{3\beta-3\beta k}.
\end{aligned}$$

The platform earns

$$\begin{aligned}
\Pi &= \int_0^1 q^*(c)\left(\frac{1-q^*(c)}{\beta}-2c\right)\,dc \\
&= \frac{k\left(4k^2-6k+3\right)}{12\beta^2}.
\end{aligned}$$

Decentralized pricing with commission-plus: Now, we will show that the optimal mechanism can be replicated with decentralized pricing. Let the platform sets its terms,

$$\phi = 1 - \frac{\gamma}{2\beta},\ w = \frac{\left(4k^2 - 6k + 3\right)(\beta - \gamma)}{6\beta^2(k-1)},$$

and assume the agents expect an average price of

$$\overline{p} = \frac{4k^2 - 6k + 3}{3\beta(1-k)}.$$

Without capacity restrictions, an agent with cost c has the following profit-maximization problem:

$$\max_p\ (1 - \beta\overline{p} + \gamma(\overline{p} - p))((1-\phi)p - c - w),$$

yielding an optimal price of

$$\begin{aligned} p^*(c, \overline{p}) &= \frac{1}{2}\left(\frac{1 - (\beta - \gamma)\overline{p}}{\gamma} + \frac{c + w}{1 - \phi}\right) \\ &= \frac{3\beta - 6\beta^2 c + 6\beta^2 ck - 6\gamma + 8\beta k^2 - 8\gamma k^2 - 9\beta k + 12\gamma k}{6\beta\gamma(k-1)}. \end{aligned} \tag{28}$$

Let $\tilde{c}$ be the highest cost agent that is bounded by capacity:

$$q(p^*(\tilde{c}, \overline{p}), \overline{p}) = k \implies \tilde{c} = \frac{1 - 2t}{2\beta}.$$

All agents with cost less than $\tilde{c}$ set the price that makes their demand exactly equal to k:

$$\begin{aligned} p^*(\tilde{c}, \overline{p}) &= \frac{1 + (\gamma - \beta)\overline{p} - k}{\gamma} \\ &= \frac{-3\gamma + \beta k^2 - 4\gamma k^2 + 6\gamma k}{3\beta\gamma k - 3\beta\gamma}. \end{aligned}$$

Others will set the price in (28). Let c_h be the highest cost agent that can participate with non-negative demand:

$$q(p^*(c_h, \overline{p}), \overline{p}) = 0 \implies c_h = \frac{1}{2\beta}.$$

The realized average price needs to be consistent with expectation:

$$\begin{aligned} \overline{p} &= \frac{\int_0^{c_h} q(p^*(c, \overline{p}), \overline{p}) p^*(c, \overline{p})\, dc}{\int_0^{c_h} q(p^*(c, \overline{p}), \overline{p})\, dc} \\ &= \frac{\int_0^{\tilde{c}} k p^*(\tilde{c}, \overline{p})\, dc + \int_{\tilde{c}}^{c_h} q(p^*(c, \overline{p}), \overline{p}) p^*(c, \overline{p})\, dc}{\int_0^{\tilde{c}} k\, dc + \int_{\tilde{c}}^{c_h} q(p^*(c, \overline{p}), \overline{p})\, dc} \\ &= \frac{4k^2 - 6k + 3}{3\beta(1-k)}. \end{aligned}$$

The platform earns

$$\Pi^{\mathcal{C}+} = \int\limits_0^{\tilde{c}} k(\phi p^*(\tilde{c}, \overline{p}) + w)\ dc + \int\limits_{\tilde{c}}^{c_h} q(p^*(c, \overline{p}), \overline{p})(\phi p^*(c, \overline{p}) + w)\ dc$$
$$= \frac{k\left(4k^2 - 6k + 3\right)}{12\beta^2},$$

same as the optimal mechanism.

□

Extension with Other Cost Distributions. We will characterize the equilibrium of the platform's profit-maximizing optimal mechanism and optimal decentralized pricing contract with a commission-plus structure assuming that agents have a probability density function $g(c)$ and cumulative distribution function $G(c)$ with support $c \in [0, 1]$.

Optimal mechanism:

The proof of the optimal mechanism remains unchanged up to the point at which the platform's problem is defined in (22). With a general set of cost distributions, the platform's problem is

$$\begin{aligned} \max_{p(c), F(c)} \quad & \int\limits_0^1 F(c) g(c)\ dc \\ \text{s.t.} \quad & \pi_c(c) \geq \pi_c(\tilde{c}), \forall c \in [0,1], \forall \tilde{c} \in [0,1] \\ & \pi_c(c) \geq 0, \forall c \in [0,1] \\ & 1 - \beta\overline{p} + \gamma(\overline{p} - p(c)) \geq 0, \forall c \in [0,1] \end{aligned}$$

with an additional requirement for the average price to be consistent with agent expectations:

$$\overline{p} = \frac{\int\limits_0^{c_h} q(p(c), \overline{p}) p(c) g(c) dc}{\int\limits_0^{c_h} q(p(c), \overline{p}) g(c) dc}. \tag{29}$$

Replacing IR and IC constraints with necessary and sufficient conditions in (20) and (21):

$$
\begin{aligned}
\max_{p(c),F(c)} \quad & \int_0^1 F(c)g(c)\, dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in [0,1] \\
& F(c) = u(c,p(c)) + \int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k, \forall c \in [0,1] \\
& 1 - \beta\overline{p} + \gamma(\overline{p} - p(c)) \geq 0, \forall c \in [0,1] \\
& \text{Eq. (29)} \\
= \max_{p(c)} \quad & \int_0^1 \left(u(c,p(c)) + \int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \right) g(c)\, dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in [0,1] \\
& 1 - \beta\overline{p} + \gamma(\overline{p} - p(c)) \geq 0, \forall c \in [0,1] \\
& \text{Eq. (29).}
\end{aligned}
$$

Using integration by parts,

$$
\begin{aligned}
\int_0^1 \left(\int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \right) g(c) dc &= \left[\left(\int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \right) G(c) \right]_0^1 - \int_0^1 \left(-\frac{\partial u(c,p(c))}{\partial c} \right) G(c)\, dc \\
&= \int_0^1 \frac{\partial u(c,p(c))}{\partial c} G(c)\, dc.
\end{aligned}
$$

The platform's problem converts to

$$
\begin{aligned}
\max_{p(c)} \quad & \int_0^1 \left(u(c,p(c)) + \frac{\partial u(c,p(c))}{\partial c} \frac{G(c)}{g(c)} \right) g(c)\, dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in [0,1] \\
& 1 - \beta\overline{p} + \gamma(\overline{p} - p(c)) \geq 0, \forall c \in [0,1] \\
& \text{Eq. (29)} \\
= \max_{p(c)} \quad & \int_0^1 \left((1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))(p(c) - c) - (1 - \beta\overline{p} + \gamma(\overline{p} - p(c))) \frac{G(c)}{g(c)} \right) g(c)\, dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in [0,1] \\
& 1 - \beta\overline{p} + \gamma(\overline{p} - p(c)) \geq 0, \forall c \in [0,1] \\
& \text{Eq. (29)} \\
= \max_{p(c)} \quad & \int_0^1 (1 - \beta\overline{p} + \gamma(\overline{p} - p(c))) \left(p(c) - c - \frac{G(c)}{g(c)} \right) g(c)\, dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in [0,1] \\
& 1 - \beta\overline{p} + \gamma(\overline{p} - p(c)) \geq 0, \forall c \in [0,1] \\
& \text{Eq. (29).}
\end{aligned}
$$

We re-formulate this as a quantity-choice problem. By Equation (1), we have one-to-one equivalence between price and quantity:

$$p(c) = \frac{1 + (\gamma - \beta)\overline{p} - q(c)}{\gamma}.$$

The average price in Equation (29) is

$$\begin{aligned}
\overline{p} &= \frac{\int_0^1 q(c) \left(\frac{1 + (\gamma - \beta)\overline{p} - q(c)}{\gamma} \right) g(c)\, dc}{\int_0^1 q(c) g(c)\, dc} \\
&= \frac{\int_0^1 \frac{q(c)\,(1 - q(c))}{\gamma} g(c)\, dc}{\int_0^1 q(c) g(c)\, dc} + \overline{p} \left(1 - \frac{\beta}{\gamma} \right),
\end{aligned}$$

which simplifies to

$$\overline{p} = \frac{\int_0^1 q(c)\,(1 - q(c))\, g(c)\, dc}{\beta \int_0^1 q(c) g(c)\, dc}. \tag{30}$$

Platform's optimal quantity-choice problem is

$$\begin{aligned}
\max_{q(c)} & \int_0^1 q(c) \left(\frac{1 + (\gamma - \beta)\overline{p} - q(c)}{\gamma} - c - \frac{G(c)}{g(c)} \right) g(c)\, dc \\
\text{s.t. } & q'(c) \leq 0, \forall c \in [0, 1] \\
& q(c) \geq 0, \forall c \in [0, 1] \\
& \text{Eq. (30).}
\end{aligned}$$

We can further simplify the objective function. Notice that Equation (30) implies:

$$\overline{p} \int_0^1 q(c) g(c)\, dc = \frac{1}{\beta} \int_0^1 q(c)\,(1 - q(c))\, g(c)\, dc.$$

Using this relationship,

$$
\begin{aligned}
\Pi &= \int_0^1 q(c)\left(\frac{1+(\gamma-\beta)\overline{p}-q(c)}{\gamma} - c - \frac{G(c)}{g(c)}\right) g(c)\ dc \\
&= \int_0^1 q(c)\left(\frac{1-q(c)}{\gamma} - c - \frac{G(c)}{g(c)}\right) g(c)\ dc + \left(\frac{\gamma-\beta}{\gamma}\right)\overline{p}\int_0^1 q(c)g(c)\ dc \\
&= \int_0^1 q(c)\left(\frac{1-q(c)}{\gamma} - c - \frac{G(c)}{g(c)}\right) g(c)\ dc + \left(\frac{\gamma-\beta}{\gamma}\right)\frac{1}{\beta}\int_0^1 q(c)\left(1-q(c)\right) g(c)\, dc \\
&= \int_0^1 \left(q(c)\left(\frac{1-q(c)}{\gamma} - c - \frac{G(c)}{g(c)}\right) + \left(\frac{\gamma-\beta}{\gamma}\right)\frac{1}{\beta} q(c)\left(1-q(c)\right)\right) g(c)\, dc \\
&= \int_0^1 q(c)\left(\frac{1-q(c)}{\beta} - c - \frac{G(c)}{g(c)}\right) g(c)\ dc.
\end{aligned}
$$

The problem terms no longer depend on the average price:

$$
\begin{aligned}
&\max_{q(c)} \int_0^1 q(c)\left(\frac{1-q(c)}{\beta} - c - \frac{G(c)}{g(c)}\right) g(c)\ dc \\
&\text{s.t. } q'(c) \le 0, \forall c \in [0,1] \\
&\qquad q(c) \ge 0, \forall c \in [0,1].
\end{aligned}
$$

Relaxing the first constraint, we can decompose the problem into individual sub-problems for all agents:

$$
\begin{aligned}
&\max_{q(c)} q(c)\left(\frac{1-q(c)}{\beta} - c - \frac{G(c)}{g(c)}\right)\ dc \\
&\text{s.t. } q(c) \ge 0.
\end{aligned}
$$

This is the maximization of a simple quadratic function with a linear constraint. The optimal quantity is:

$$
q^*(c) = \max\left\{0, \frac{1}{2} - \beta\frac{G(c)}{g(c)} - \beta c\right\}.
$$

The solution to this relaxed problem is optimal for the platform's optimal mechanism only if the relaxed constraint is satisfied. That requires the following regularity condition on the cost distribution:

$$
\begin{aligned}
\frac{dq^*(c)}{dc} \le 0 &\iff \frac{d}{dc}\left(\frac{1}{2} - \beta\frac{G(c)}{g(c)} - \beta c\right) \le 0 \\
&\iff \frac{G(c)g'(c)}{g(c)^2} - 2 \le 0 \ \ \forall c \in [0,1].
\end{aligned}
$$

The highest cost with non-zero quantity does not have a closed form solution, but can evaluated numerically as an implicit solution to

$$q^*(\hat{c}) = \frac{1}{2} - \beta\frac{G(\hat{c})}{g(\hat{c})} - \beta\hat{c} = 0.$$

The platform's profit under the optimal mechanism is

$$\Pi = \int_0^1 q^*(c)\left(\frac{1-q^*(c)}{\beta} - c - \frac{G(c)}{g(c)}\right) g(c)\; dc.$$

Decentralized pricing with commission-plus:

In decentralized pricing with commission-plus, conditional on participation, an agent with cost c setting a price p earns

$$\pi_c(p,\overline{p}) = (1-\beta\overline{p}+\gamma(\overline{p}-p))((1-\phi)p - w - c).$$

Each agent chooses the price that maximizes own earnings. From (10),

$$p^*(c,\overline{p}) \quad = \frac{1}{2}\left(\frac{1-(\beta-\gamma)\overline{p}}{\gamma} + \frac{c+w}{1-\phi}\right).$$

The agent earns

$$\pi_c(p^*(c,\overline{p}),\overline{p}) = \frac{(1-\phi+\overline{p}(1-\phi)(\gamma-\beta)-\gamma(c+w))^2}{4\gamma(1-\phi)}.$$

Agent earnings are decreasing in cost, c, and therefore there exists a threshold cost, c_h, for which agent with cost c participates if and only if $c \le c_h$. Define c_0 to be the lowest cost that yields zero profit:

$$\pi_{c_0}(p^*(c_0,\overline{p}),\overline{p}) = 0 \implies c_0 = c_0(\phi,w) = \frac{(1-\phi)(1+(\gamma-\beta)\overline{p})}{\gamma} - w,$$

Then,

$$c_h = c_h(\phi,w) = \min\{1, c_0\}.$$

The average market price is defined by:

$$\overline{p} = \frac{\int_0^{c_h} q(p(c),\overline{p})p(c)g(c)dc}{\int_0^{c_h} q(p(c),\overline{p})g(c)dc} = \frac{\int_0^{\min\left\{1,\frac{(1-\phi)(1+(\gamma-\beta)\overline{p})}{\gamma}-w\right\}} \left(\frac{(1+\overline{p}(\gamma-\beta))^2}{4\gamma} - \frac{\gamma(c+w)^2}{4(1-\phi)^2}\right) g(c)dc}{\int_0^{\min\left\{1,\frac{(1-\phi)(1+(\gamma-\beta)\overline{p})}{\gamma}-w\right\}} \frac{1}{2}\left(1+\overline{p}(\gamma-\beta) - \frac{\gamma(c+w)}{1-\phi}\right) g(c)dc}. \tag{31}$$

The uniqueness of a solution to (31) is not guaranteed, and identifying conditions needed on the cost distribution that ensure uniqueness is nontrivial. As a result, we only work with distributions with known analytical forms. Proposition 2 shows the uniqueness for a uniform distribution. It is also possible to show uniqueness for Beta(2,2), Beta(3,3), Beta(4,4) and Beta(5,5) distributions. This involves following the same steps as in Proposition 2: plugging in $g(c)$, formulating right-hand-side as a piece-wise function of c_0 (rather than $\overline{p}$) and showing monotonicity. For brevity, we omit the details here.

The platform's profit-maximization problem is:

$$\begin{aligned}\max_{\phi,w} \quad \Pi^{\mathcal{D}}(\phi,w) &= (\phi\overline{p}+w)\int_0^{c_h} q(p^*(c,\overline{p}),\overline{p})g(c)\,dc \\ &= \frac{1}{2}(\phi\overline{p}+w)\int_0^{c_h}\left(1+\overline{p}(\gamma-\beta)-\frac{\gamma(c+w)}{1-\phi}\right)g(c)dc.\end{aligned}$$

There is no closed-form solution to the platform's problem for a general $q(c)$; however, for any given distribution with known analytical form, it is possible to conduct a numerical search across the (ϕ, w) space to identify the platform's optimal profit.

□

Extension with Throughput-Maximization. In the quantity maximizing optimal contract, let $p(c)$ be the price the platform assigns to agent c and $F(c)$ be the fee collected. By Proposition 5, Equations (20) and (21) are necessary and sufficient conditions for agents' IR and IC constraints. In any truth-inducing mechanism, the equilibrium fees charged to the agent who reports a cost c is

$$F(c) = u(c,p(c)) + \int_c^1 \frac{\partial u(c_k,p(c_k))}{\partial c_k}\,dc_k.$$

The total quantity served in the market is

$$\int_0^1 q(c)\,dc = \int_0^1 (1-\beta\overline{p}+\gamma(\overline{p}-p(c)))\,dc.$$

The optimal truth-inducing contract that maximizes total quantity served subject to non-negative profit constraint is characterized through the following problem:

$$\begin{aligned}
\max_{p(c)} \quad & \int_0^1 (1-\beta\overline{p}+\gamma(\overline{p}-p(c)))\, dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in [0,1] \\
& 1-\beta\overline{p}+\gamma(\overline{p}-p(c)) \geq 0, \forall c \in [0,1] \\
& F(c) = u(c,p(c)) + \int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k, \forall c \in [0,1] \\
& \int_0^1 F(c)\, dc \geq 0 \\
& \text{Eq. (2)} \\
= \max_{p(c)} \quad & \int_0^1 (1-\beta\overline{p}+\gamma(\overline{p}-p(c)))\, dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in [0,1] \\
& 1-\beta\overline{p}+\gamma(\overline{p}-p(c)) \geq 0, \forall c \in [0,1] \\
& \int_0^1 (1-\beta\overline{p}+\gamma(\overline{p}-p(c)))(p(c)-2c)\, dc \geq 0 \\
& \text{Eq. (2)},
\end{aligned}$$

where the equivalence of the constraints follow from Equations (23) and (24). We reformulate the problem with quantities as decision variables:

$$\begin{aligned}
\max_{q(c)} \quad & \int_0^1 q(c)\, dc \\
\text{s.t.} \quad & q'(c) \leq 0, \forall c \in [0,1] \\
& q(c) \geq 0, \forall c \in [0,1] \\
& \int_0^1 q(c) \left(\frac{1+(\gamma-\beta)\overline{p}-q(c)}{\gamma} - 2c \right) dc \geq 0 \\
& \text{Eq. (25)}.
\end{aligned}$$

Following the same steps as Equation (26), we transform the objective function such that it does not depend on $\overline{p}$:

$$\begin{aligned}
\max_{q(c)} \quad & \int_0^1 q(c)\, dc \\
\text{s.t.} \quad & q'(c) \leq 0, \forall c \in [0,1] \\
& q(c) \geq 0, \forall c \in [0,1] \\
& \int_0^1 q(c) \left(\frac{1-q(c)}{\beta} - 2c \right) dc \geq 0.
\end{aligned}$$

To find the solution to this problem, let us look at a related problem, where the platform maximizes the sum of its profits and total quantity served, with latter weighted by some $\lambda > 0$:

$$
\begin{aligned}
\max_{q(c)} \quad & \int_0^1 q(c) \left(\frac{1 - q(c)}{\beta} - 2c \right) dc + \lambda \int_0^1 q(c)\, dc \\
\text{s.t.} \quad & q'(c) \le 0, \forall c \in [0,1] \\
& q(c) \ge 0, \forall c \in [0,1] \\
= \max_{q(c)} \quad & \int_0^1 q(c) \left(\frac{1 - q(c)}{\beta} + \lambda - 2c \right) dc \\
\text{s.t.} \quad & q'(c) \le 0, \forall c \in [0,1] \\
& q(c) \ge 0, \forall c \in [0,1]
\end{aligned}
\tag{32}
$$

Relaxing the first constraint, we can decompose this into individual sub-problems for all agents:

$$
\begin{aligned}
& \max_{q(c)} q(c) \left(\frac{1 - q(c)}{\beta} + \lambda - 2c \right) dc \\
& \text{s.t. } q(c) \ge 0.
\end{aligned}
$$

This is the maximization of a simple quadratic function with a lower-bound. The optimal quantity is:

$$
q^*(c) = \max \left\{ 0, \frac{1}{2}(1 + \beta\lambda) - \beta c \right\}.
$$

This solution also satisfies:

$$
q^{*\prime}(c) \le 0.
$$

Therefore, the solution to this relaxed problem is also optimal for (32).

The highest cost agent with non-zero quantity is

$$
c_h = \min \left\{ 1, \frac{1 + \beta\lambda}{2\beta} \right\}.
$$

We will now show that, $(1 + \beta\lambda)/(2\beta) < 1$ for all λ values that yield non-negative profits for the platform as long as $\beta > 1$ restriction discussed in Section 3 holds. To prove by contradiction, assume $(1 + \beta\lambda)/(2\beta) \ge 1$ and $c_h = 1$ in equilibrium. The platform earns

$$
\Pi = \int_0^1 q^*(c) \left(\frac{1 - q(c)}{\beta} - 2c \right) dc = \frac{1}{12} \left(\beta \left(4 - 3\lambda^2 \right) + \frac{3}{\beta} - 6 \right).
$$

The profit is non-negative only if

$$
\lambda \le \sqrt{\frac{1}{\beta^2} - \frac{2}{\beta} + \frac{4}{3}}.
$$

This implies

$$
\frac{1 + \beta\lambda}{2\beta} = \frac{1 + \beta\sqrt{\frac{1}{\beta^2} - \frac{2}{\beta} + \frac{4}{3}}}{2\beta} < 1
$$

for all feasible $\beta > 1$, creating a contradiction. This means that $(1+\beta\lambda)/(2\beta) \geq 1$ cannot be true and that we have

$$c_h = \frac{1+\beta\lambda}{2\beta}.$$

The profits earned by the platform is

$$\Pi = \int_0^{c_h} q^*(c)\left(\frac{1-q(c)}{\beta} - 2c\right)\, dc = \frac{(1+\beta\lambda)^2(1-2\beta\lambda)}{24\beta^2}.$$

By definition, Equation (32) maximizes the total quantity served subject to the constraint that the platform earns

$$\Pi = \frac{(1+\beta\lambda)^2(1-2\beta\lambda)}{24\beta^2}.$$

Total throughput-maximizing solution is then the one that yields exactly zero profits for the platform:

$$\Pi = \frac{(1+\beta\lambda^*)^2(1-2\beta\lambda^*)}{24\beta^2} = 0 \iff \lambda^* = \frac{1}{2\beta},$$

which yields the total quantity served of

$$\int_0^{c_h} q^*(c)\, dc = \frac{(1+\beta\lambda^*)^2}{8\beta} = \frac{9}{32\beta}.$$

Under this optimal mechanism, the highest cost that participates and the average market price are:

$$\overline{p} = \frac{1}{2\beta},$$
$$c_h = \frac{3}{4\beta}.$$

Total quantity of customers served in the market is

$$Q = \frac{9}{32\beta}.$$

The fee platform charges to agent with cost c is

$$F^*(c) = \frac{(\gamma-2\beta)(4\beta c-3)(4\beta c-1)}{32\beta\gamma}.$$

We will now show that the platform can implement this mechanism with commission-plus. With decentralized pricing, let the platform sets its terms,

$$\phi = 1 - \frac{\gamma}{2\beta},\ w = \frac{\gamma-2\beta}{4\beta^2},$$

and assume agents expect an average price of

$$\overline{p} = \frac{1}{2\beta}.$$

By Equation (10), an agent with cost c posts a price under decentralized pricing

$$\begin{aligned} p^*(c, \overline{p}) &= \frac{1}{2}\left(\frac{1-(\beta-\gamma)\overline{p}}{\gamma} + \frac{c+w}{1-\phi}\right) \\ &= \frac{2\gamma - \beta + 4\beta^2 c}{4\beta\gamma}. \end{aligned}$$

Let c_h be the highest cost agent that can participate with non-negative demand:

$$q(p^*(c_h, \overline{p}), \overline{p}) = 0 \implies c_h = \frac{3}{4\beta}.$$

The realized average price is characterized by Equation (17),

$$\overline{p} = \frac{2(1-\phi) + \gamma w}{(1-\phi)(2\beta+\gamma)} = \frac{1}{2\beta}.$$

The platform's profit is

$$\begin{aligned} \Pi^{\mathcal{C}+} &= \int_0^{c_h} q(p^*(c, \overline{p}), \overline{p})(\phi p^*(c, \overline{p}) + w)\, dc \\ &= 0. \end{aligned}$$

Total quantity served in the market is

$$\begin{aligned} Q &= \int_0^{c_h} q(p^*(c, \overline{p}), \overline{p})\, dc \\ &= \frac{9}{32\beta}, \end{aligned}$$

same as the optimal mechanism.

□

C Robustness to Straight Average Utility Model

With the quadratic customer utility model and continuum of agents, a representative customer's utility is

$$U(q) = \frac{\alpha}{\gamma}\int_0^1 q(c)\, dc - \frac{1}{2\gamma}\int_0^1 q(c)^2\, dc - \frac{\beta}{2\gamma}\left(\int_0^1 q(c)\, dc\right)^2 - \int_0^1 q(c)p(c)\, dc.$$

The finite agent version of this utility model was introduced in Shubik and Levitan (1980) and the linear demand model that arises from this utility function has been used by many authors (e.g., Abhishek et al. 2016, Hagiu and Wright 2019b, Inderst and Shaffer 2019). Others like Melitz and Ottaviano (2008), Foster et al. (2008), Altomonte et al. (2016) use the continuum agent version above, for which the demand of an agent with positive quantity is

$$q(p(c), Q) = \alpha - \beta Q - \gamma p(c) \tag{33}$$

where

$$Q = \int_0^1 q(p(c), Q)\, dc.$$

We establish the robustness of our qualitative results to the choice of customer utility model in this document by replicating our analyses using the demand model specified in (33) and observing that our main qualitative results continue to hold. More specifically, we show the following results.

- When platform uses fees that are linear in price, there is a unique centralized pricing equilibrium (C.1.1) and a unique decentralized pricing equilibrium (Section C.1.3).
- With just a simple commission contract, centralized pricing can be better or worse than decentralized pricing depending on the realization of demand parameters (Section C.1.5).
- Platform's optimal mechanism can be implemented with decentralized pricing using either a commission-plus contract (Section C.1.7) or a quantity discount contract (Section C.1.8).
- Decentralized pricing with commission-plus replicates the optimal mechanism even when the agents have capacity constraints (Section C.1.13) or the platform adopts throughput maximization as its objective (Section C.1.14).

C.1 Analysis

C.1.1 Centralized Pricing

In centralized pricing, there exists a single price in the market. Let p be the price platform sets and ϕ be the portion of revenue retained. Conditional on participation, an agent with cost c earns

$$\pi_c(p) = (\alpha - \beta Q - \gamma p)((1 - \phi)p - c).$$

Agent earnings decrease in cost, and therefore there exists a threshold cost, c_h, for which agent with cost c participates if and only if $c \leq c_h$. The highest cost that participates is

$$\pi_{c_h}(p) = 0 \implies c_h = c_h(p, \phi) = (1 - \phi)p.$$

The total quantity served in the market is

$$
\begin{aligned}
Q &= \int_0^{\hat{c}} (\alpha - \beta Q - \gamma p)\, dc \\
&= \hat{c}(\alpha - \beta Q - \gamma p) \\
&= (1-\phi)p(\alpha - \beta Q - \gamma p).
\end{aligned}
$$

Solving for Q, we get

$$Q = \frac{p(1-\phi)(\alpha - \gamma p)}{1 + \beta p(1-\phi)}.$$

Plugging this back into the demand function:

$$q(p) = \frac{\alpha - \gamma p}{1 + \beta(1-\phi)p}.$$

The demand is non-negative only if $p \leq \alpha/\gamma$.

The platform's profit-maximization problem is:

$$
\begin{aligned}
\max_{\phi, p} \quad \Pi^{\mathcal{C}}(p,\phi) &= \phi \int_0^{c_h(\phi)} q(p,Q)p\, dc \\
&= \frac{p^2(1-\phi)\phi(\alpha - \gamma p)}{\alpha + \beta p(1-\phi)}.
\end{aligned}
$$

The problem is concave in ϕ within the feasible region $0 \leq \phi \leq 1$, $0 \leq p \leq \alpha/\gamma$, e.g.,

$$\frac{\partial^2 \Pi^{\mathcal{C}}(p,\phi)}{\partial \phi^2} = -\frac{2p^2(1+\beta p)(\alpha - \gamma p)}{(1+\beta p(1-\phi))^3} < 0,$$

and therefore, platform's optimal commission is defined by the first order condition

$$\frac{\partial \Pi^{\mathcal{C}}(p,\phi)}{\partial \phi} = 0 \implies \frac{p^2(\alpha - \gamma p)\left(\beta p(\phi-1)^2 - 2\phi + 1\right)}{(\beta p(\phi - 1) - 1)^2} = 0.$$

This gives a unique feasible solution,

$$\phi^*(p) = \frac{1 + \beta p - \sqrt{1+\beta p}}{\beta p}.$$

Plugging this back into optimization problem and looking at the total derivative with respect to price, we observe

$$
\begin{aligned}
\frac{d\Pi^{\mathcal{C}}(p,\phi^*(p))}{dp} \geq 0 &\iff \frac{\alpha\left(\beta - \frac{\beta}{\sqrt{1+\beta p}}\right) + \gamma\left(-2\beta p + 3\sqrt{1+\beta p} - \frac{1}{\sqrt{1+\beta p}} - 2\right)}{\beta^2} \geq 0 \\
&\iff p \geq \frac{1}{8\beta}\sqrt{\frac{8\alpha\beta + 9\gamma}{\gamma}} + \frac{4\alpha\beta - 3\gamma}{8\beta\gamma}.
\end{aligned}
$$

Hence, platform's profit function is quasi-concave in price and is maximized at

$$p^* = \frac{1}{8\beta}\sqrt{\frac{8\alpha\beta + 9\gamma}{\gamma}} + \frac{4\alpha\beta - 3\gamma}{8\beta\gamma}.$$

Plugging in the optimal decisions, the platform's equilibrium profit is

$$\Pi^{\mathcal{C}} = \frac{8\alpha^2\beta^2 - 9\gamma^{3/2}\sqrt{8\alpha\beta + 9\gamma} + 36\alpha\beta\gamma - 8\alpha\beta\sqrt{\gamma}\sqrt{8\alpha\beta + 9\gamma} + 27\gamma^2}{32\beta^3\gamma}.$$

C.1.2 Decentralized Pricing

With decentralized pricing, agents select their own price to post on the platform. A price equilibrium occurs when all agents select optimal prices given the equilibrium prices of others.

First, in Section C.1.3, we show that the existence and uniqueness of a decentralized pricing equilibrium are guaranteed for a set of contracts where the fees agents pay to the platform are linear in the price they select. Then, in Section C.1.4, we characterize the decentralized pricing equilibrium assuming that the platform uses a commission only contract.

C.1.3 The Existence And Uniqueness Of Decentralized Pricing Equilibrium

Consider a broad class of fee structures such that an agent pays the platform (i) a base unit fee, w, per unit served and (ii) a commission, ϕp, per unit served where $\phi < 1$ is the fixed commission rate and p is the agent's price. An agent with demand q pays the platform in total $q\,(w + \phi p)$. The fixed per-unit fee, w, can be negative, meaning that it is actually a per-unit subsidy.

With this payment structure, we seek to determine the uniqueness of a price equilibrium. Each agent's earnings depend on the total quantity on the platform, so it is sufficient to consider a pricing equilibrium in which each agent expects the total quantity to be Q_e and all agents select prices based on that expectation that yield Q_e as the actual total quantity served.

An agent with total quantity expectation Q_e, cost c and price p expects to earn

$$\begin{aligned}\pi_c(p, Q_e) &= q(p, Q_e)\,((1-\phi)p - c - w) \\ &= (\alpha - \beta Q_e - \gamma p)\,((1-\phi)\,p - c - w)\,.\end{aligned}$$

Because earnings are strictly concave in price, there exists a unique price, $p^*(c, Q_e)$, that maximizes the agent's earnings,

$$p^*(c, Q_e) = \frac{(\alpha - \beta Q_e)}{2\gamma} + \frac{c + w}{2(1-\phi)}. \tag{34}$$

Let the total quantity realized be Q. This agent receives $q(p^*(c, Q_e), Q)$ demand,

$$q(p^*(c, Q_e), Q) = \frac{1}{2}\left(\alpha + \beta(Q_e - 2Q) - \frac{\gamma(c + w)}{1-\phi}\right)^+. \tag{35}$$

The demand is decreasing in cost, c. Define c_0 to be the smallest cost such that the optimal quantity is zero,

$$c_0 = \min\{c : q(p^*(c, Q_e), Q) = 0\} = \frac{\alpha(1-\phi) + \beta(1-\phi)(Q_e - 2Q)}{\gamma} - w. \tag{36}$$

The realized total quantity is

$$Q = \int_0^{\min\{c_0,1\}} q(p^*(c, Q_e), Q)\, dc. \tag{37}$$

An equilibrium exists if $Q_e = Q$. We will show that there can exist only one such equilibrium in any market.

From (36), given an equilibrium, i.e., $Q = Q_e$, the total quantity in equilibrium can be expressed in terms of c_0,

$$Q = \frac{\alpha(1-\phi) - \gamma(c_0 + w)}{\beta(1-\phi)}. \tag{38}$$

From (34), (35), (38) and $Q_e = Q$, the equilibrium condition (37) can be written as

$$\begin{aligned} 0 &= Q - \int_0^{\min\{c_0,1\}} q(p^*(c, Q), Q)\, dc \\ &= \frac{\alpha(1-\phi) - \gamma(c_0 + w)}{\beta(1-\phi)} - \int_0^{\min\{c_0,1\}} \frac{1}{2}\left(\alpha - \beta Q - \frac{\gamma(c+w)}{1-\phi}\right) dc \qquad (39) \\ \iff 0 &= \begin{cases} \dfrac{4\alpha(1-\phi) - \beta\gamma c_0^2 - 4\gamma(c_0 + w)}{4\beta(1-\phi)} & \text{, if } c_0 \le 1, \\ \dfrac{\gamma(\beta - 2(\beta+2)c_0 - 4w) - 4\alpha(\phi - 1)}{4\beta(1-\phi)} & \text{, if } c_0 > 1. \end{cases} \end{aligned}$$

The right-hand side of (39) is continuous at $c_0 = 1$. It is also monotonically decreasing in c_0:

$$\frac{\partial}{\partial c_0}\left(\frac{4\alpha(1-\phi) - \beta\gamma c_0^2 - 4\gamma(c_0 + w)}{4\beta(1-\phi)}\right) = -\frac{\gamma(\beta c_0 + 2)}{2\beta(1-\phi)} < 0\ \forall c_0 \le 1,$$
$$\frac{\partial}{\partial c_0}\left(\frac{\gamma(\beta - 2(\beta+2)c_0 - 4w) - 4\alpha(\phi - 1)}{4\beta(1-\phi)}\right) = -\frac{(\beta+2)\gamma}{2\beta(1-\phi)} < 0\ \forall c_0 > 1.$$

As a result, the right-hand side of (39) only cross zero once, yielding a unique c_0.

The unique feasible candidate c_0 in the domain $c_0 \le 1$ is

$$c_0 = \frac{2\left(\sqrt{\gamma(\gamma + \beta(\alpha(1-\phi) - \gamma w))} - \gamma\right)}{\beta\gamma}. \tag{40}$$

From (38) and (40), for this candidate c_0, there exists a unique equilibrium total quantity,

$$Q = \frac{\alpha\beta(1-\phi) + \gamma(2 - \beta w) - 2\sqrt{\gamma(\gamma + \beta(\alpha(1-\phi) - \gamma w))}}{\beta^2(1-\phi)}. \tag{41}$$

C.1.4 Decentralized Pricing With Just A Commission Fee

The analysis in Section C.1.3 shows that the uniqueness of a total quantity that satisfies consistent expectations equilibrium is guaranteed. In decentralized pricing with commission fees, conditional on participation, an agent with cost c setting a price p earns

$$\pi_c(p, Q) = (\alpha - \beta Q - \gamma p)((1 - \phi)p - c).$$

Agent's profit depends on individual price, p, and also the total quantity served in the market, Q, which is characterized in the equilibrium.

Each agent chooses the price that maximizes own earnings. From (34),

$$p^*(c, Q) = \frac{\alpha - \beta Q}{2\gamma} + \frac{c + w}{2(1 - \phi)}.$$

The agent earns

$$\pi_c(p, Q) = \frac{((\alpha - Q\beta)(1 - \phi) - \gamma c)^2}{4\gamma(1 - \phi)}.$$

Agent earnings are decreasing in cost, c, and therefore there exists a threshold cost, c_h, for which agent with cost c participates if and only if $c \leq c_h$. Define c_0 to be the lowest cost that yields zero profit:

$$\pi_{c_0}(p^*(c_0, Q), Q) = 0 \implies c_0 = c_0(\phi) = \frac{(\alpha - Q\beta)(1 - \phi)}{\gamma}.$$

Then,

$$c_h = c_h(\phi) = \min\{1, c_0\}.$$

With $c_0 \leq 1$, the total quantity is uniquely defined by (41):

$$Q = \frac{\alpha\beta(1 - \phi) + \gamma(2 - \beta w) - 2\sqrt{\gamma(\gamma + \beta(\alpha(1 - \phi) - \gamma w))}}{\beta^2(1 - \phi)}.$$

The platform's profit-maximization problem is:

$$\begin{aligned}\max_{\phi} \quad \Pi^{\mathcal{D}}(\phi) &= \phi \int_0^{c_h(\phi)} q(p^*(c, Q), Q)p^*(c, Q)\, dc \\ &= \frac{4\phi\left(\sqrt{\gamma(\alpha\beta(1 - \phi) + \gamma)} - \gamma\right)^3}{3\beta^3\gamma^2(1 - \phi)^2}.\end{aligned}$$

The profit takes positive values for all $0 < \phi < 1$ and is zero at both boundary points, $\phi = 0$ and $\phi = 1$. Because the profit function is continuous, any maximizer is an interior point and defined by the first order condition:

$$\frac{\partial \Pi^{\mathcal{D}}(\phi)}{\partial \phi} = \frac{2\left(\gamma - \sqrt{\gamma(\alpha\beta(1 - \phi) + \gamma)}\right)^2 \left(2(\phi + 1)\left(\sqrt{\gamma(\alpha\beta(1 - \phi) + \gamma)} - \gamma\right) - \alpha\beta(\phi - 2)(\phi - 1)\right)}{3\beta^3\gamma(\phi - 1)^3\sqrt{\gamma(\alpha\beta(1 - \phi) + \gamma)}} = 0. \tag{42}$$

(42) can be shown to yield a unique solution. Note that for any $0 < \phi < 1$,

$$\frac{2\left(\gamma - \sqrt{\gamma(\alpha\beta(1-\phi)+\gamma)}\right)^2}{3\beta^3\gamma(\phi-1)^3\sqrt{\gamma(\alpha\beta(1-\phi)+\gamma)}} < 0.$$

This means that FOC is satisfied when

$$\begin{aligned} & 2(\phi^*+1)\left(\sqrt{\gamma(\alpha\beta(1-\phi^*)+\gamma)}-\gamma\right) - \alpha\beta(\phi^*-2)(\phi^*-1) = 0 \\ \iff & \frac{2}{\phi^*-1}\left(\sqrt{\gamma(\alpha\beta(1-\phi^*)+\gamma)}-\gamma\right) - \alpha\beta\left(\frac{\phi^*-2}{\phi^*+1}\right) = 0. \end{aligned} \tag{43}$$

The left-hand side of the equality is monotonically increasing in ϕ^* within the range $\phi^* \in [0, 1]$, and therefore can only cross zero once. Hence, (43) uniquely defines platform's optimal commission, ϕ^*.

C.1.5 Comparison of Centralized and Decentralized Pricing

Equation (43) corresponds to a root of a cubic polynomial. While Cardano's formula can find closed-form expressions for roots of a cubic polynomial, the expressions are length. To compute platform's profit under decentralized pricing and compare it with centralized pricing, we use a computer algebra system and find that there exists a $k \approx 0.149$ such that

$$\Pi^{\mathcal{D}} > \Pi^{\mathcal{C}} \iff \gamma < k\alpha\beta.$$

That is, a platform may prefer centralized or decentralized pricing depending on the competition characteristics of the market. More specifically, the platform's profit is higher with decentralized pricing in markets with low γ, and it is higher with centralized pricing in markets with high γ.

C.1.6 Optimal Mechanism

According to the Revelation Principle, the set of optimal mechanisms can be found within the set of truth-inducing mechanisms. In such a mechanism, the platform posts a menu that maps each cost to a price and a fee. Each agent reports a cost and is assigned the price and fee associated with that cost, and each agent (*i*) earns non-negative profits (Individual Rationality), and (*ii*) prefers to report own cost truthfully given that all other agents do the same (Incentive Compatibility). Following this structure, let $p(c)$ be the price the platform assigns to agent c and let $F(c)$ be the fee collected. Note that $F(c)$ is the total fee and not per-unit, $F(c) = q(p(c), Q)f(c)$. Let $\pi_c(\tilde{c})$ be an agent's earnings with cost c when the agent reports cost $\tilde{c}$:

$$\pi_c(\tilde{c}) = u(c, p(\tilde{c})) - F(\tilde{c})$$

where

$$u(c, p(\tilde{c})) = (\alpha - \beta Q - \gamma p(\tilde{c}))\,(p(\tilde{c}) - c).$$

The Individual Rationality (IR) and the Incentive Compatibility (IC) constraints are

$$\pi_c(c) \geq 0,$$
$$\pi_c(c) \geq \pi_c(\tilde{c}),$$

respectively for all $c \in [0,1]$, $\tilde{c} \in [0,1]$.

Because of the IC constraints, agent earnings are strictly decreasing in cost:

$$\begin{aligned}\pi_c(c) \geq \pi_c(\tilde{c}) &= (\alpha - \beta Q - \gamma p(\tilde{c}))\,(p(\tilde{c}) - c) - F(\tilde{c}) \\ &> (\alpha - \beta Q - \gamma p(\tilde{c}))\,(p(\tilde{c}) - \tilde{c}) - F(\tilde{c}) = \pi_{\tilde{c}}(\tilde{c})\end{aligned}$$

for all $\tilde{c} > c$. This also implies that the agent with the highest cost, $c = 1$, earns zero under the optimal truth-inducing mechanism: if the highest cost agent were to earn a strictly positive amount, then the platform could uniformly increase the fee, $F(\tilde{c})$, for all participating agents, thus increasing its profit.

We can re-formulate an agent's earnings as

$$\pi_c(\tilde{c}) = \pi_{\tilde{c}}(\tilde{c}) - u(\tilde{c}, p(\tilde{c})) + u(c, p(\tilde{c})).$$

For a pair of agents with costs c, $\tilde{c}$, IC constraints imply

$$\begin{aligned}\pi_c(c) \geq \pi_c(\tilde{c}) &= \pi_{\tilde{c}}(\tilde{c}) - u(\tilde{c}, p(\tilde{c})) + u(c, p(\tilde{c})), \\ \pi_{\tilde{c}}(\tilde{c}) \geq \pi_{\tilde{c}}(c) &= \pi_c(c) - u(c, p(c)) + u(\tilde{c}, p(c)).\end{aligned}$$

These inequalities can be combined:

$$\begin{aligned}&u(\tilde{c}, p(c)) - u(c, p(c)) \leq \pi_{\tilde{c}}(\tilde{c}) - \pi_c(c) \leq u(\tilde{c}, p(\tilde{c})) - u(c, p(\tilde{c})) \\ \iff &\int_c^{\tilde{c}} \frac{\partial u(c_k, p(c))}{\partial c_k}\, dc_k \leq \pi_{\tilde{c}}(\tilde{c}) - \pi_c(c) \leq \int_c^{\tilde{c}} \frac{\partial u(c_k, p(\tilde{c}))}{\partial c_k}\, dc_k.\end{aligned} \tag{44}$$

This inequality has two important consequences. First, ignoring the middle term, Equation (44) implies

$$
\begin{aligned}
&\int_{c}^{\tilde{c}} \frac{\partial u(c_k, p(c))}{\partial c_k}\, dc_k \leq \int_{c}^{\tilde{c}} \frac{\partial u(c_k, p(\tilde{c}))}{\partial c_k}\, dc_k,\ \forall c \in [0,1], \forall \tilde{c} \in [0,1] \\
\iff & -\int_{c}^{\tilde{c}} (\alpha - \beta Q - \gamma p(c))\, dc_k \leq -\int_{c}^{\tilde{c}} (\alpha - \beta Q - \gamma p(\tilde{c}))\, dc_k,\ \forall c \in [0,1], \forall \tilde{c} \in [0,1] \\
\iff & (\alpha - \beta Q - \gamma p(c))\,(c - \tilde{c}) \leq (\alpha - \beta Q - \gamma p(\tilde{c}))\,(c - \tilde{c}),\ \forall c \in [0,1], \forall \tilde{c} \in [0,1] \\
\iff & (p(c) - p(\tilde{c}))(c - \tilde{c}) \geq 0,\ \forall c \in [0,1], \forall \tilde{c} \in [0,1] \\
\iff & p'(c) \geq 0,\ \forall c \in [0,1].
\end{aligned}
\tag{45}
$$

Second, because $u(\tilde{c}, p(c))$ is a continuous function of $\tilde{c}$, this inequality implies that $\pi_{\tilde{c}}(\tilde{c})$ is also continuous with respect to the Lebesgue measure and thus almost everywhere differentiable. Furthermore, its derivative is

$$\frac{d\pi_c(c)}{dc} = \frac{\partial u(c, p(c))}{\partial c}.$$

Using the fundamental theorem of calculus:

$$\pi_{\tilde{c}}(\tilde{c}) = \pi_c(c) + \int_{c}^{\tilde{c}} \frac{d\pi_{c_k}(c_k)}{dc_k}\, dc_k = \pi_c(c) + \int_{c}^{\tilde{c}} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k.$$

Because the highest cost agent earns zero, setting $\tilde{c} = 1$, the equation simplifies to

$$\pi_c(c) = -\int_{c}^{1} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k,$$

which means

$$F(c) = u(c, p(c)) + \int_{c}^{1} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k. \tag{46}$$

Our analysis so far has established that (45) and (46) are necessary conditions for IR and IC. Now, we will show that they are also sufficient. By establishing an equivalence between the two sets of constraints, we will be able to replace the IR and IC constraints in the platform's optimal mechanism problem with Equations (45) and (46) and therefore convert the problem into a tractable form.

It is straightforward to show that (46) is sufficient to imply IR:

$$\begin{aligned}
\pi_c(c) &= u(c,p(c)) - F(c) \\
&= u(c,p(c)) - u(c,p(c)) - \int_c^1 \frac{\partial u(c_k,p(c_k))}{\partial c_k}\, dc_k \\
&= -\int_c^1 \frac{\partial u(c_k,p(c_k))}{\partial c_k}\, dc_k \\
&= \int_c^1 (\alpha - \beta Q - \gamma p(c_k))\, dc_k \geq 0\ \forall c \in [0,1].
\end{aligned}$$

Similarly for IC, we have

$$\begin{aligned}
&\pi_c(\tilde{c}) - \pi_c(c) = u(c,p(\tilde{c})) - F(\tilde{c}) - u(c,p(c)) + F(c) \\
=&u(c,p(\tilde{c})) - u(\tilde{c},p(\tilde{c})) - \int_{\tilde{c}}^1 \frac{\partial u(c_k,p(c_k))}{\partial c_k}\, dc_k - u(c,p(c)) + u(c,p(c)) + \int_c^1 \frac{\partial u(c_k,p(c_k))}{\partial c_k}\, dc_k \\
=&u(c,p(\tilde{c})) - u(\tilde{c},p(\tilde{c})) - \int_{\tilde{c}}^c \frac{\partial u(c_k,p(c_k))}{\partial c_k}\, dc_k \\
=&\int_{\tilde{c}}^c \frac{\partial u(c_k,p(\tilde{c}))}{\partial c_k}\, dc_k - \int_{\tilde{c}}^c \frac{\partial u(c_k,p(c_k))}{\partial c_k}\, dc_k \\
=&\int_{\tilde{c}}^c (\alpha - \beta Q - \gamma p(\tilde{c}))\, dc_k - \int_{\tilde{c}}^c (\alpha - \beta Q - \gamma p(c_k))\, dc_k \\
=&\gamma \int_{\tilde{c}}^c (p(c_k) - p(\tilde{c}))\, dc_k \geq 0,\ \forall c \in [0,1], \tilde{c} \in [0,1].
\end{aligned}$$

Now, we construct the platform's problem. The platform chooses $p(c)$ and $F(c)$ to maximize total profits subject to IR and IC constraints, and the natural restriction that the demand of an agent cannot be negative:

$$\begin{aligned}
\max_{p(c),F(c)} \quad & \int_0^1 F(c)\, dc \\
\text{s.t.} \quad & \pi_c(c) \geq \pi_c(\tilde{c}), \forall c \in [0,1], \forall \tilde{c} \in [0,1] \\
& \pi_c(c) \geq 0, \forall c \in [0,1] \\
& \alpha - \beta Q - \gamma p(c) \geq 0, \forall c \in [0,1] \\
& Q = \int_0^1 (\alpha - \beta Q - \gamma p(c))\, dc.
\end{aligned}$$

Using the equivalence we found earlier, we replace IR and IC constraints with Equations (45) and (46):

$$\begin{aligned}
&\max_{p(c),F(c)} && \int_0^1 F(c)\, dc \\
&\text{s.t.} && p'(c) \geq 0, \forall c \in [0,1] \\
& && F(c) = u(c,p(c)) + \int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k, \forall c \in [0,1] \\
& && \alpha - \beta Q - \gamma p(c) \geq 0, \forall c \in [0,1] \\
& && Q = \int_0^1 (\alpha - \beta Q - \gamma p(c))\, dc \\
= &\max_{p(c)} && \int_0^1 \left(u(c,p(c)) + \int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \right) dc \\
&\text{s.t.} && p'(c) \geq 0, \forall c \in [0,1] \\
& && \alpha - \beta Q - \gamma p(c) \geq 0, \forall c \in [0,1] \\
& && Q = \int_0^1 (\alpha - \beta Q - \gamma p(c))\, dc.
\end{aligned}$$

Using integration by parts,

$$\begin{aligned}
\int_0^1 \left(\int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \right) dc &= \left[\left(\int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \right) c \right]_0^1 - \int_0^1 \left(-\frac{\partial u(c,p(c))}{\partial c} \right) c\, dc \\
&= \int_0^1 \frac{\partial u(c,p(c))}{\partial c} c\, dc.
\end{aligned} \tag{47}$$

The platform's problem converts to

$$
\begin{aligned}
&\max_{p(c)} && \int_0^1 \left(u(c,p(c)) + \frac{\partial u(c,p(c))}{\partial c} c \right) \, dc \\
&\text{s.t.} && p'(c) \geq 0, \forall c \in [0,1] \\
& && \alpha - \beta Q - \gamma p(c) \geq 0, \forall c \in [0,1] \\
& && Q = \int_0^1 (\alpha - \beta Q - \gamma p(c)) \, dc \\
= &\max_{p(c)} && \int_0^1 \Big((\alpha - \beta Q - \gamma p(c))(p(c) - c) - (\alpha - \beta Q - \gamma p(c)) c \Big) \, dc \\
&\text{s.t.} && p'(c) \geq 0, \forall c \in [0,1] \\
& && \alpha - \beta Q - \gamma p(c) \geq 0, \forall c \in [0,1] \\
& && Q = \int_0^1 (\alpha - \beta Q - \gamma p(c)) \, dc \\
= &\max_{p(c)} && \int_0^1 (\alpha - \beta Q - \gamma p(c))(p(c) - 2c) \, dc \\
&\text{s.t.} && p'(c) \geq 0, \forall c \in [0,1] \\
& && \alpha - \beta Q - \gamma p(c) \geq 0, \forall c \in [0,1] \\
& && Q = \int_0^1 (\alpha - \beta Q - \gamma p(c)) \, dc
\end{aligned} \tag{48}
$$

We re-formulate this as a quantity-choice problem. Let $q(c)$ be the demand platform assigns to agent with cost c. We have one-to-one equivalence between price and quantity:

$$p(c) = \frac{\alpha - \beta Q - q(c)}{\gamma}.$$

Plugging this back into the platform's problem and expanding the Q expression, we have the following problem

$$
\begin{aligned}
&\max_{q(c)} \frac{\alpha}{\gamma} \int_0^1 q(c) \, dc - \frac{1}{\gamma} \int_0^1 q(c)^2 \, dc - \frac{\beta}{\gamma} \left(\int_0^1 q(c) \, dc \right)^2 - \int_0^1 q(c) 2c \, dc \\
&\text{s.t. } q'(c) \leq 0, \forall c \in [0,1] \\
&\quad\;\; q(c) \geq 0, \forall c \in [0,1].
\end{aligned} \tag{49}
$$

Relaxing the first constraint, the problem is a specific case of quadratic utility maximization where the prices of the agents are equal to twice their costs, $p(c) = 2c$. For all agents with non-zero assigned quantity, we have

$$q(c) = \frac{\alpha}{2} - \beta Q - \gamma c \tag{50}$$

where

$$Q = \int_0^1 q(c) \, dc. \tag{51}$$

This solution also satisfies the relaxed contraint. Therefore, the solution in (50) is also optimal for the platform's optimal mechanism.

Solving for (50) and (51) simultaneously, we get one feasible total quantity,

$$Q = \frac{\alpha\beta + 2\gamma - 2\sqrt{\alpha\beta\gamma + \gamma^2}}{2\beta^2}.$$

Other equilibrium characteristics are as follows:

$$\begin{aligned}
p^*(c) &= \frac{\alpha - \beta Q - q(c)}{\gamma} \\
&= \frac{\alpha}{2\gamma} + c \\
F^*(c) &= u(c, p^*(c)) + \int_c^1 \frac{\partial u(c_k, p^*(c_k))}{\partial c_k}\, dc_k \\
&= \frac{\alpha\beta\left(\sqrt{\gamma(\alpha\beta+\gamma)} - \gamma(\beta c + 2)\right) - \gamma\left(\gamma\left(\beta^2c^2 + 2\beta c + 2\right) - 2(\beta c + 1)\sqrt{\gamma(\alpha\beta+\gamma)}\right)}{2\beta^2\gamma}.
\end{aligned}$$

With the optimal mechanism, we have

$$\begin{aligned}
F^*(c) &= \frac{\alpha\beta\left(\sqrt{\gamma(\alpha\beta+\gamma)} - \gamma(\beta c + 2)\right) - \gamma\left(\gamma\left(\beta^2c^2 + 2\beta c + 2\right) - 2(\beta c + 1)\sqrt{\gamma(\alpha\beta+\gamma)}\right)}{2\beta^2\gamma} \\
&= q^*(c)\left(\frac{1}{2}p^*(c) + \frac{-2\sqrt{\gamma(\alpha\beta+\gamma)} + \alpha\beta + 2\gamma}{4\beta\gamma}\right).
\end{aligned}$$

Because we can map the fee structure of the optimal mechanism to one that is linear in price, the uniqueness arguments in Section C.1.3 also apply to the optimal mechanism. Therefore, with the optimal mechanism, there exists a unique equilibrium total quantity.

The platform earns

$$\begin{aligned}
\Pi &= \frac{\alpha}{\gamma}\int_0^1 q(c)\, dc - \frac{1}{\gamma}\int_0^1 q(c)^2\, dc - \frac{\beta}{\gamma}\left(\int_0^1 q(c)\, dc\right)^2 - 2\int_0^1 q(c)c\, dc \\
&= \frac{\left(-2\sqrt{\gamma(\alpha\beta+\gamma)} + 3\alpha\beta + 2\gamma\right)\left(\gamma - \sqrt{\gamma(\alpha\beta+\gamma)}\right)^2}{12\beta^3\gamma^2}.
\end{aligned}$$

C.1.7 Replicating The Optimal Mechanism With Commission and Per-unit Fee Contract

With the optimal mechanism, there is a unique candidate commission and per-unit fee structure that may yield the same equilibrium outcome as the optimal mechanism. To show that this structure indeed replicates the optimal mechanism, let the platform sets its terms

$$\phi = \frac{1}{2},\ w = \frac{-2\sqrt{\gamma(\alpha\beta+\gamma)} + \alpha\beta + 2\gamma}{4\beta\gamma},$$

and assume agents expect a total quantity of

$$Q = \frac{\alpha\beta + 2\gamma - 2\sqrt{\alpha\beta\gamma + \gamma^2}}{2\beta^2}.$$

By Equation (34), an agent with cost c posts a price under commission-plus

$$\begin{aligned} p^*(c, Q) &= \frac{\alpha - \beta Q}{2\gamma} + \frac{c + w}{2(1-\phi)} \\ &= \frac{\alpha}{2\gamma} + c. \end{aligned}$$

The price chosen by any agent is exactly equivalent to the one in the optimal mechanism. The same is true for the fees paid. Because prices and fees are the only decision variables in the setting, then commission-plus with the given terms replicates the optimal mechanism.

C.1.8 Replicating The Optimal Mechanism With Quantity Discount Contract

With the optimal mechanism, we have

$$\begin{aligned} F^*(c) &= \frac{\alpha\beta\left(\sqrt{\gamma(\alpha\beta+\gamma)} - \gamma(\beta c + 2)\right) - \gamma\left(\gamma\left(\beta^2 c^2 + 2\beta c + 2\right) - 2(\beta c + 1)\sqrt{\gamma(\alpha\beta+\gamma)}\right)}{2\beta^2\gamma} \\ &= q^*(c)\left(-\frac{1}{2\gamma}q^*(c) + \frac{\alpha}{2\gamma}\right). \end{aligned}$$

There is a unique candidate quantity pricing fee structure that may yield the same equilibrium outcome as the optimal mechanism. To show that this structure indeed replicates the optimal mechanism, let the platform sets its terms

$$\phi = -\frac{1}{2\gamma}, \; w = \frac{\alpha}{2\gamma}.$$

and assume agents expect a total quantity of

$$Q = \frac{\alpha\beta + 2\gamma - 2\sqrt{\alpha\beta\gamma + \gamma^2}}{2\beta^2}.$$

Conditional on participation, an agent with cost c setting a price p earns

$$\pi_c(p, Q) = (\alpha - \beta Q - \gamma p)(p - c - \phi(\alpha - \beta Q - \gamma p) - w).$$

Because earnings are strictly concave in price, there exists a unique price, $p^*(c, Q)$, that maximizes the agent's earnings,

$$\begin{aligned} p^*(c, Q) &= \frac{2\alpha\gamma\phi + \alpha + c\gamma - 2\beta\gamma Q\phi - \beta Q + \gamma w}{2\gamma(\gamma\phi + 1)} \\ &= \frac{\alpha}{2\gamma} + c. \end{aligned}$$

The price chosen by any agent is exactly equivalent to the one in the optimal mechanism. Same is true for the fees paid. Because prices and fees are the only decision variables in the setting, then quantity pricing with the given terms replicates the optimal mechanism.

C.1.9 Capacity Constraints

We will now solve the equilibria of centralized pricing, decentralized pricing, and optimal mechanism under the assumption that agents have a maximum capacity to serve k demand.

C.1.10 Centralized Pricing

Without capacity constraints, the number of customers served by each agent under centralized pricing is

$$q(p,Q) = \frac{\sqrt{\gamma}(2\gamma-\alpha\beta)\left(-\sqrt{\gamma}\sqrt{8\alpha\beta+9\gamma}+4\alpha\beta+3\gamma\right)}{\beta\left(2\alpha\beta\left(\sqrt{8\alpha\beta+9\gamma}+3\sqrt{\gamma}\right)+\gamma\left(\sqrt{8\alpha\beta+9\gamma}+13\sqrt{\gamma}\right)\right)}.$$

With capacity constraints, the capacity will be binding if and only if k is smaller than this amount. When capacity is binding, if platform sets a price p, the profit agent with cost c earns is

$$\pi_c(p) = k((1-\phi)p - c)$$

Agents participate as long as they earn positive profits. The highest cost that participates, c_h, earns zero

$$\pi_{c_h}(p) = 0 \implies c_h = c_h(p,\phi) = (1-\phi)p.$$

The platform's profit is:

$$\begin{aligned}
\Pi^{\mathcal{C}}(p,\phi) &= \int_0^{c_h(p,\phi)} k\phi p \, dc \\
&= k\phi(1-\phi)p^2.
\end{aligned}$$

The objective is monotonically increasing in p. It is optimal for the platform to increase the price as long as there is excess demand. In the optimal, quantity served should equal to the capacity:

$$\begin{aligned}
k &= \alpha - \beta Q - \gamma p \\
&= \alpha - \beta k c_h - \gamma p \\
&= \alpha - \beta k(1-\phi)p - \gamma p
\end{aligned}$$

yielding a price

$$p = \frac{\alpha - k}{\gamma + \beta k(1-\phi)}.$$

Plugging this into the objective,

$$\Pi^{\mathcal{C}}(\phi) = \frac{k(1-\phi)\phi(k-\alpha)^2}{(\gamma + k\beta(1-\phi))^2}.$$

Within the feasible set of parameters, we have

$$\begin{aligned}\frac{\partial \Pi^{\mathcal{C}}(\phi)}{\partial \phi} =& \frac{k(k-\alpha)^2(\gamma(2\phi-1)+\beta k(\phi-1))}{(\beta k(\phi-1)-\gamma)^3} \geq 0 \\ \iff& \phi \leq \frac{\gamma+\beta k}{2\gamma+\beta k}.\end{aligned}$$

Therefore, the platform's profit is quasi-concave in ϕ and is maximized at

$$\phi^* = \frac{\gamma+\beta k}{2\gamma+\beta k}.$$

The platform's equilibrium profit is

$$\Pi^{\mathcal{C}} = \frac{k(\alpha-k)^2}{4\gamma(\gamma+\beta k)}.$$

C.1.11 Decentralized Pricing

Without capacity constraints, the highest number of customers an agent serves under decentralized pricing with just commission is

$$q(p^*(0,Q),Q) = \frac{\sqrt{\gamma(\alpha\beta(1-\phi^*)+\gamma)}-\gamma}{\beta(1-\phi^*)}.$$

where ϕ^* is given by (43).

With capacity constraints, the capacity will be binding if and only if k is smaller than this amount. The agents need to incorporate the capacity restriction in their pricing decision. Profit earned by agent with cost c and price p conditional on participation is

$$\pi_c(p,Q) = \min\{\alpha-\beta Q+\gamma p, k\}((1-\phi)p-c).$$

Agent sets the price that maximizes own profits:

$$\max_p \min\{\alpha-\beta Q+\gamma p, k\}((1-\phi)p-c).$$

Without capacity constraints, an agent's optimal price is characterized by Equation (34),

$$p^*(c,Q) = \frac{\alpha-\beta Q}{2\gamma} + \frac{c}{2(1-\phi)},$$

which gives quantity

$$q(p^*(c,Q),Q) = \frac{1}{2}\left(\alpha-\beta Q-\frac{\gamma c}{1-\phi}\right).$$

Since the quantity served is decreasing in cost, there exists a threshold, $\tilde{c}$, where only the agents with costs $c \leq \tilde{c}$ are constrained in capacity. The threshold agent has exactly the quantity k:

$$q(p^*(\tilde{c},Q),Q) = k \implies \tilde{c} = \tilde{c}(\phi) = \frac{(1-\phi)(\alpha-\beta Q-2k)}{\gamma}.$$

If an agent is capacity constrained, $k \leq q(p^*(c,Q),Q)$, any decrease in price greater than the one that matches capacity and demand decreases the agent's margin, but does not increase the quantity served. Hence, an agent never sets a price lower than the capacity binding level. The optimal price of an agent with cost $c < \tilde{c}$ is:

$$p^*(\tilde{c},Q) = \frac{\alpha - \beta Q - k}{\gamma}.$$

The highest cost agent that participates has 0 demand:

$$q(p^*(c_h,Q),Q) = 0 \implies c_h = c_h(\phi) = \frac{(1-\phi)(\alpha-\beta Q)}{\gamma}.$$

Agents' equilibrium prices are

$$p^*(c,Q) = \begin{cases} \dfrac{\alpha-\beta Q-k}{\gamma}, & c \leq \tilde{c}, \\ \dfrac{\alpha-\beta Q}{2\gamma} + \dfrac{c}{2(1-\phi)}, & \tilde{c} < c \leq c_h. \end{cases}$$

Agents with a cost greater than c_h cannot profitably participate.

The total quantity is defined in the equilibrium:

$$\begin{aligned} Q &= \int_0^{c_h} q(p^*(c,Q),Q)\; dc \\ &= \int_0^{\tilde{c}} k\; dc + \int_{\tilde{c}}^{c_h} \frac{1}{2}\left(\alpha - \beta Q - \frac{\gamma c}{1-\phi}\right)\; dc \\ &= \frac{k(1-\phi)(\alpha-\beta Q-k)}{\gamma}. \end{aligned}$$

There is a unique total quantity:

$$Q = \frac{k(1-\phi)(\alpha-k)}{\gamma + \beta k(1-\phi)}.$$

Platform's profit maximization problem is:

$$\begin{aligned} \max_{\phi} \quad \Pi^{\mathcal{D}}(\phi) &= \phi \int_0^{c_h} q(p^*(c,Q),Q)p^*(c,Q)\; dc \\ &= \phi\left(\int_0^{\tilde{c}} k\frac{\alpha-\beta Q-k}{\gamma}\; dc + \int_0^{\tilde{c}} \tfrac{1}{2}\left(\alpha-\beta Q - \tfrac{\gamma c}{1-\phi}\right)\left(\frac{\alpha-\beta Q}{2\gamma} + \frac{c}{2(1-\phi)}\right)\right) \\ &= \frac{k(1-\phi)\phi\left(3\alpha^2\gamma^2 + \beta^2 k^4(\phi-1)^2 - 2\beta\gamma k^3(\phi-1) + 4\gamma^2k^2 - 6\alpha\gamma^2 k\right)}{3\gamma^2(\gamma + k(\beta-\beta\phi))^2}. \end{aligned}$$

The profit takes positive values for all $0 < \phi < 1$, $k > 0$ and is zero at both boundary points, $\phi = 0$ and $\phi = 1$. Because the profit function is continuous, any maximizer is an interior point and defined by the first order condition:

$$\frac{\partial \Pi^{\mathcal{D}}(\phi)}{\partial \phi} = \frac{k}{3\gamma^2(\gamma + k(\beta - \beta\phi))^3}\Bigg(3\alpha^2\gamma^3(1-2\phi) + \beta^3 k^5(\phi-1)^3(2\phi-1) - 3\beta^2\gamma k^4(\phi-1)^2(2\phi-1)$$
$$+6\beta\gamma^2k^3(\phi-1)^2 + 2\gamma^2k^2(3\alpha\beta(\phi-1)+\gamma(2-4\phi)) - 3\alpha\gamma^2 k(\alpha\beta(\phi-1)+\gamma(2-4\phi))\Bigg) = 0, \tag{52}$$

Equation (52) can be shown to yield a unique solution. Note that for any $0 < \phi < 1$,

$$\frac{k}{3\gamma^2(\gamma + k\beta(1-\phi))^3} > 0.$$

So, FOC is satisfied when

$$\begin{aligned} 3\alpha^2\gamma^3(1-2\phi^*) + \beta^3k^5(\phi^*-1)^3(2\phi^*-1) - 3\beta^2\gamma k^4(\phi^*-1)^2(2\phi^*-1) + 6\beta\gamma^2k^3(\phi^*-1)^2 \\ +2\gamma^2k^2(3\alpha\beta(\phi^*-1)+\gamma(2-4\phi^*)) - 3\alpha\gamma^2k(\alpha\beta(\phi^*-1)+\gamma(2-4\phi^*)) = 0. \end{aligned} \tag{53}$$

The left-hand side of the equality increases monotonically in ϕ^* within the feasible range $\phi^* \in [0,1]$, $\tilde{c}(\phi^*) \geq 0$, and therefore the expression can only cross zero once. Hence, (53) uniquely defines platform's optimal commission, ϕ^*.

C.1.12 Optimal Mechanism

Without capacity constraints, the highest number of customers an agent serves under the optimal mechanism is

$$q^*(0) = \frac{\sqrt{\gamma(\alpha\beta+\gamma)} - \gamma}{\beta}.$$

The capacity is binding if and only if k is smaller than this amount.

Our proof for the platform's capacity-unconstrained optimal mechanism is applicable up until we reach Equation (49). In the presence of demand constraints, platform's optimal mechanism problem has an additional constraint:

$$\begin{aligned} \max_{q(c)} \ & \frac{\alpha}{\gamma}\int_0^1 q(c)\,dc - \frac{1}{\gamma}\int_0^1 q(c)^2\,dc - \frac{\beta}{\gamma}\left(\int_0^1 q(c)\,dc\right)^2 - \int_0^1 q(c)2c\,dc \\ \text{s.t. } & q'(c) \leq 0, \forall c \in [0,1] \\ & q(c) \geq 0, \forall c \in [0,1] \\ & q(c) \leq k, \forall c \in [0,1]. \end{aligned}$$

Ignoring the first constraint, this is a quadratic utility maximization problem with capacity constraints in a market where agent prices are equal to twice their cost, $p(c) = 2c$. For all agents whose quantities are not effected by the two constraints, the assigned optimal quantity is

$$q(c) = \frac{\alpha}{2} - \beta Q - \gamma c.$$

where

$$.Q = \int_0^1 q(c)\, dc \tag{54}$$

For all others, the quantity constraints are binding. That is, the quantity assigned to an agent with cost c is

$$q(c) = \min\left\{ \left(\frac{\alpha}{2} - \beta Q - \gamma c\right)^+, k \right\} \tag{55}$$

This solution also satisfies the relaxed contraint. Therefore, the solution to the relaxed problem is also optimal for the platform's optimal mechanism.

Solving for (54) and (55) simultaneously, we get one feasible total quantity,

$$Q = \frac{k(\alpha - k)}{2(\gamma + \beta k)}.$$

The price assigned to agents with excess capacity is

$$\begin{aligned} p^*(c) &= \frac{\alpha - \beta Q - q(c)}{\gamma} \\ &= \frac{\alpha}{2\gamma} + c, \end{aligned}$$

capacity constrained agents are assigned the price

$$p^*(c) = \frac{(\alpha - k)(2\gamma + \beta k)}{2\gamma(\gamma + \beta k)}.$$

The platform earns

$$\begin{aligned} \Pi &= \frac{\alpha}{\gamma} \int_0^1 q(c)\, dc - \frac{1}{\gamma} \int_0^1 q(c)^2\, dc - \frac{\beta}{\gamma} \left(\int_0^1 q(c)\, dc \right)^2 - \int_0^1 q(c) 2c\, dc \\ &= \frac{k\left(3\alpha^2\gamma + \beta k^3 + 4\gamma k^2 - 6\alpha\gamma k\right)}{12\gamma^2(\gamma + \beta k)}. \end{aligned}$$

C.1.13 Decentralized Pricing With Commission And Per-Unit Fee

To show that decentralized pricing with commission-plus replicates the optimal mechanism, let the platform sets its terms

$$\phi = \frac{1}{2},\ w = \frac{\alpha\beta k - \beta k^2}{4\gamma(\gamma + \beta k)},$$

and assume agents expect a total quantity of

$$Q = \frac{k(\alpha - k)}{2(\gamma + \beta k)}.$$

Without capacity restrictions, an agent with cost c has the following profit-maximization problem:

$$\max_{p} \, (\alpha - \beta Q - \gamma p)((1 - \phi)p - c - w),$$

yielding an optimal price of

$$\begin{aligned} p^*(c, Q) &= \frac{\alpha - \beta Q}{2\gamma} + \frac{c + w}{2(1 - \phi)} \\ &= \frac{\alpha}{2\gamma} + c. \end{aligned} \tag{56}$$

Let $\tilde{c}$ be the highest cost agent that is bounded by capacity:

$$q(p^*(\tilde{c}, Q), Q) = k \implies \tilde{c} = \frac{\alpha\gamma - \beta k^2 - 2\gamma k}{2\gamma(\gamma + \beta k)}.$$

All agents with cost less than $\tilde{c}$ set the price that makes their demand exactly equal to k:

$$\begin{aligned} p^*(\tilde{c}, Q) &= \frac{\alpha - \beta Q - k}{\gamma} \\ &= \frac{(\alpha - k)(2\gamma + \beta k)}{2\gamma(\gamma + \beta k)}. \end{aligned}$$

Others will set the price in (56). Let c_h be the highest cost agent that can participate with non-negative demand:

$$q(p^*(c_h, Q), Q) = 0 \implies c_h = \frac{\alpha\gamma + \beta k^2}{2\gamma(\gamma + \beta k)}.$$

The total quantity needs to be consistent with expectation:

$$\begin{aligned} Q &= \int_0^{c_h} q(p^*(c, Q), Q) \, dc \\ &= \frac{k(\alpha - k)}{2(\gamma + \beta k)}. \end{aligned}$$

The platform earns

$$\begin{aligned} \Pi^{\mathcal{C}+} &= \int_0^{\tilde{c}} k(\phi p^*(\tilde{c}, Q) + w) \, dc + \int_{\tilde{c}}^{c_h} q(p^*(c, Q), Q)(\phi p^*(c, Q) + w) \, dc \\ &= \frac{k\left(3\alpha^2\gamma + \beta k^3 + 4\gamma k^2 - 6\alpha\gamma k\right)}{12\gamma^2(\gamma + \beta k)}, \end{aligned}$$

same as the optimal mechanism.

C.1.14 Throughput Maximization

In the quantity maximizing optimal contract, let $p(c)$ be the price the platform assigns to agent c and $F(c)$ be the fee collected. By Section C.1.6, Equations (45) and (46) are necessary and sufficient conditions for agents' IR and IC constraints. In any truth-inducing mechanism, the equilibrium fees charged to the agent who reports a cost c is

$$F(c) = u(c, p(c)) + \int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k.$$

The total quantity served in the market is

$$\int_0^1 q(c)\, dc = \int_0^1 (\alpha - \beta Q - \gamma p(c))\, dc.$$

The optimal truth-inducing contract that maximizes total quantity served subject to non-negative profit constraint is characterized through the following problem:

$$\begin{aligned}
\max_{p(c)} \quad & \int_0^1 (\alpha - \beta Q - \gamma p(c))\, dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in [0,1] \\
& \alpha - \beta Q - \gamma p(c) \geq 0, \forall c \in [0,1] \\
& F(c) = u(c, p(c)) + \int_c^1 \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k, \forall c \in [0,1] \\
& \int_0^1 F(c)\, dc \geq 0 \\
& Q = \int_0^1 (\alpha - \beta Q - \gamma p(c))\, dc \\
= \max_{p(c)} \quad & \int_0^1 (\alpha - \beta Q - \gamma p(c))\, dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in [0,1] \\
& \alpha - \beta Q - \gamma p(c) \geq 0, \forall c \in [0,1] \\
& \int_0^1 (\alpha - \beta Q - \gamma p(c))(p(c) - 2c)\, dc \geq 0 \\
& Q = \int_0^1 (\alpha - \beta Q - \gamma p(c))\, dc,
\end{aligned}$$

where the equivalence of the constraints follow from Equations (47) and (48). We reformulate the problem with quantities as decision variables:

$$
\begin{aligned}
\max_{q(c)} \quad & \int_0^1 q(c)\, dc \\
\text{s.t.} \quad & q'(c) \leq 0, \forall c \in [0,1] \\
& q(c) \geq 0, \forall c \in [0,1] \\
& \int_0^1 q(c) \left(\frac{\alpha - \beta Q - q(c)}{\gamma} - 2c \right) dc \geq 0 \\
& Q = \int_0^1 (\alpha - \beta Q - \gamma p(c))\, dc.
\end{aligned}
$$

Let us first look at a related problem, where the platform maximizes a combination of its earning and total quantity served, where the latter is weighted by λ:

$$
\begin{aligned}
\max_{q(c)} \quad & \int_0^1 q(c) \left(\frac{\alpha - \beta Q - q(c)}{\gamma} - 2c \right) dc + \lambda \int_0^1 q(c)\, dc \\
\text{s.t.} \quad & q'(c) \leq 0, \forall c \in [0,1] \\
& q(c) \geq 0, \forall c \in [0,1] \\
& Q = \int_0^1 (\alpha - \beta Q - \gamma p(c))\, dc.
\end{aligned}
$$

By modifying the value of λ, we can assign different weights to how much platform prioritizes maximizing the quantity served relative to the profit. Specifically, we can recover the quantity maximizing solution by solving for the largest feasible λ^* that makes optimal profit non-negative.

Expanding the Q expression, the problem becomes

$$
\begin{aligned}
\max_{q(c)} \quad & \left(\frac{\alpha}{\gamma} + \lambda \right) \int_0^1 q(c)\, dc - \frac{1}{\gamma} \int_0^1 q(c)^2\, dc - \frac{\beta}{\gamma} \left(\int_0^1 q(c)\, dc \right)^2 - \int_0^1 q(c) 2c\, dc \\
\text{s.t.} \quad & q'(c) \leq 0, \forall c \in [0,1] \\
& q(c) \geq 0, \forall c \in [0,1].
\end{aligned}
$$

Relaxing the first constraint, the problem is a specific case of quadratic utility maximization where the prices of the agents are equal to twice their costs, $p(c) = 2c$. For all agents with non-zero assigned quantity, we have

$$
q(c) = \frac{\alpha + \lambda\gamma}{2} - \beta Q - \gamma c \tag{57}
$$

where

$$
Q = \int_0^1 q(c)\, dc. \tag{58}
$$

This solution also satisfies the relaxed constraint. Therefore, the solution in (57) is also optimal for the platform's optimal mechanism.

Solving for (57) and (58) simultaneously, we get one feasible total quantity,

$$Q = \frac{\left(\sqrt{\gamma(\alpha\beta + \beta\gamma\lambda + \gamma)} - \gamma\right)^2}{2\beta^2\gamma}.$$

The total profit earned is

$$\begin{aligned}
\Pi &= \frac{\alpha}{\gamma}\int_0^1 q(c)\,dc - \frac{1}{\gamma}\int_0^1 q(c)^2\,dc - \frac{\beta}{\gamma}\left(\int_0^1 q(c)\,dc\right)^2 - \int_0^1 q(c)2c\,dc \\
&= \frac{\left(-2\sqrt{\gamma(\alpha\beta + \beta\gamma\lambda + \gamma)} + 3\alpha\beta + \gamma(2 - 3\beta\lambda)\right)\left(\gamma - \sqrt{\gamma(\alpha\beta + \beta\gamma\lambda + \gamma)}\right)^2}{12\beta^3\gamma^2}.
\end{aligned}$$

This expression approaches negative infinity as λ goes to infinity. To get the quantity-maximizing contract, we look for the greatest feasible λ that makes the profit non-negative:

$$\lambda^* = \frac{9\alpha\beta + 8\gamma}{9\beta\gamma} - \frac{2\sqrt{2}}{9\beta}\sqrt{\frac{9\alpha\beta + 8\gamma}{\gamma}}.$$

The total quantity served under the optimal mechanism is

$$Q = \frac{13\gamma + 9\alpha\beta - \sqrt{2}\sqrt{\gamma(9\alpha\beta + 8\gamma)} - 3\sqrt{\gamma\left(-2\sqrt{2}\sqrt{\gamma(9\alpha\beta + 8\gamma)} + 18\alpha\beta + 17\gamma\right)}}{9\beta^2}.$$

With the optimal mechanism, we have

$$\begin{aligned}
p^*(c) &= \frac{\sqrt{\frac{18\alpha\beta}{\gamma} + 16} - 4}{9\beta} + c \\
F^*(c) &= \frac{1}{54\beta^2}\Bigg(10\sqrt{\gamma\left(-2\sqrt{2}\sqrt{\gamma(9\alpha\beta + 8\gamma)} + 18\alpha\beta + 17\gamma\right)} \\
&\quad + 2\sqrt{2}\sqrt{(9\alpha\beta + 8\gamma)\left(-2\sqrt{2}\sqrt{\gamma(9\alpha\beta + 8\gamma)} + 18\alpha\beta + 17\gamma\right)} - 54\gamma \\
&\quad - 3\beta\left(18\alpha + c\left(2\sqrt{2}\sqrt{\gamma(9\alpha\beta + 8\gamma)} - 6\sqrt{\gamma\left(-2\sqrt{2}\sqrt{\gamma(9\alpha\beta + 8\gamma)} + 18\alpha\beta + 17\gamma\right)} + \gamma(9\beta c + 10)\right)\right)\Bigg) \\
&= q^*(c)\left(\frac{1}{2}p^*(c) + \frac{\sqrt{2}\sqrt{\gamma(9\alpha\beta + 8\gamma)} - 3\sqrt{\gamma\left(-2\sqrt{2}\sqrt{\gamma(9\alpha\beta + 8\gamma)} + 18\alpha\beta + 17\gamma\right)} + 5\gamma}{18\beta\gamma}\right).
\end{aligned}$$

Now, let us show that any throughput-maximizing optimal contract can be replicated with a commission-plus contract.

Let the platform sets its terms

$$\phi = \frac{1}{2},\ w = \frac{\sqrt{2}\sqrt{\gamma(9\alpha\beta+8\gamma)} - 3\sqrt{\gamma\left(-2\sqrt{2}\sqrt{\gamma(9\alpha\beta+8\gamma)} + 18\alpha\beta + 17\gamma\right)} + 5\gamma}{18\beta\gamma},$$

and assume agents expect a total quantity of

$$Q = \frac{13\gamma + 9\alpha\beta - \sqrt{2}\sqrt{\gamma(9\alpha\beta+8\gamma)} - 3\sqrt{\gamma\left(-2\sqrt{2}\sqrt{\gamma(9\alpha\beta+8\gamma)} + 18\alpha\beta + 17\gamma\right)}}{9\beta^2}.$$

By Equation (34), an agent with cost c posts a price under commission-plus

$$\begin{aligned} p^*(c,Q) &= \frac{\alpha - \beta Q}{2\gamma} + \frac{c+w}{2(1-\phi)} \\ &= \frac{\sqrt{\frac{18\alpha\beta}{\gamma} + 16} - 4}{9\beta} + c. \end{aligned}$$

The price chosen by any agent is exactly equivalent to the one in the optimal mechanism. The same is true for the fees paid. Because prices and fees are the only decision variables in the setting, commission-plus with the given terms replicates the optimal mechanism.